The 12th Japan–Vietnam Joint Seminar on Commutative Algebra

September 8–12, 2026

Nha Trang, Khanh Hoa, Vietnam

United Nations Educational, Scientific and Cultural Organization

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This is the booklet for the 12th Japan–Vietnam Joint Seminar on Commutative Algebra.

Further information can be found at:

https://math.ac.vn/conference/JV2026

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About

The 12th Japan–Vietnam Joint Seminar on Commutative Algebra

This is the 12th seminar in a series of joint meetings on commutative algebra between Japan and Vietnam. Since its beginning, the seminar has been guided by the motto *"By and for young mathematicians."* The aim of the series is to provide a regular forum for commutative algebraists from both countries, to foster collaboration among participants, and to help nurture the next generation of researchers in the field.

Dates

September 8–12, 2026

Venue

Meeting Room No. 3

University of Khanh Hoa

01 Nguyen Chanh, Nha Trang ward

Khanh Hoa province, Vietnam

Organizing Institutions

University of Khanh Hoa (host)

Institute of Mathematics – VAST

International Center for Research and Training in Mathematics

Organizing Committee

Doan Trung Cuong
Tran Quang Hoa
Le Thanh Nhan
Naoyuki Matsuoka
Toshinori Kobayashi
Kazuho Ozeki
Phan Phien
Phan Duc Ngai

Advisory Committee

Nguyen Tu Cuong
Ngo Viet Trung
Kei-ichi Watanabe

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Tuesday, September 8

Strongly G-regular local rings

Ryo Takahashi

Nagoya University

A local ring R is called *G-regular* if every finitely generated Gorenstein projective R-module (i.e., totally reflexive R-module) is projective (equivalently, free). In this talk, we consider a variant of this notion in which the finite generation assumption is removed. To be more precise, we say that R is *strongly G-regular* if every (possibly infinitely generated) Gorenstein projective R-module is projective. It is clear that strong G-regularity implies G-regularity, and it is natural to ask whether the converse also holds. We relate this question to the condition that the residue field of R is *finitely built* (in the sense of Dwyer–Greenlees–Iyengar) out of R and the dualizing complex of R. This talk is based on ongoing joint work with Kaito Kimura, Yuki Mifune and Yuya Otake.

Syzygy categories and Auslander-type conditions

Yuya Otake

The University of Tokyo

Auslander and Bass began the study of syzygy categories in the 1960s. In particular, the theories of Auslander–Bridger and Evans–Griffith showed that grade conditions on Ext modules are closely related to important properties of syzygies, namely higher torsionfreeness. In this talk, we explain how Auslander-type grade conditions are related to Serre's (S_n) conditions. If time permits, we will also present a result on the problem, studied by Auslander and Reiten, of determining when an extension of syzygies is again a syzygy.

On the decomposition of syzygy modules over Golod local rings

Toshinori Kobayashi

Meiji University

This talk is based on joint work [1, 2].

Let $(R, \mathfrak{m}, k)$ be a Noetherian local ring with embedding dimension e. For a finitely generated R-module M, let $\mathrm{syz}_n^R(M)$ denote the nth syzygy module in a minimal free resolution of M. Understanding direct-sum decompositions of syzygy modules is one of the central topics in commutative algebra. Recently, in joint work with D. T. Cuong, H. Dao, D. Eisenbud, C. Polini and B. Ulrich, we proved a decomposition theorem for the syzygy module $\mathrm{syz}_{e+1}^R(k)$ over Golod rings.

In this talk, I will explain this theorem, review the necessary background, and present our recent work on its converse.

References

1. D. T. Cuong, H. Dao, D. Eisenbud, T. Kobayashi, C. Polini, B. Ulrich, *Syzygies of the residue field over Golod rings*, arXiv:2408.13425.
2. D. T. Cuong, T. Kobayashi, *On direct summands of syzygies of the residue field of a local ring*, arXiv:2510.24220.

Sequentially Cohen-Macaulay defect of modules over standard graded k-algebras

Tran Thi Hoang Anh

Thanh Dong University

This is based on joint works with Doan Trung Cuong and Nguyen Dang Hop. In this report, we study a sequentially Cohen–Macaulay defect for finitely generated graded modules over standard graded k-algebras. We begin with the polynomial case $R = k[x_1, \ldots, x_n]$, where the deficiency modules of a graded R-module M are defined by

$$K^i(M) = \operatorname{Ext}_R^{n-i}(M, \omega_R), \qquad \omega_R \cong R(-n).$$

We then extend this construction to a standard graded k-algebra $R' = R/I$ by using a homogeneous polynomial presentation $R \twoheadrightarrow R'$. The main object is the invariant

$$\operatorname{scmd}_R^*(M) = \max\{i - \operatorname{depth}_R K_R^i(M) \mid K_R^i(M) \neq 0\},$$

which measures how far M is from being sequentially Cohen–Macaulay. We discuss its basic properties in the graded setting, with particular attention to polynomial extensions and tensor products of standard graded k-algebras.

Quasi-factoriality of monoid algebras in Laurent polynomials

Takanori Nagamine

Nihon University

Let R be a domain and $R[x_1^{\pm 1}, \ldots, x_n^{\pm 1}]$ the Laurent polynomial ring over R. In this talk, we study quasi-factorially closed (qfc) R-subalgebras of $R[x_1^{\pm 1}, \ldots, x_n^{\pm 1}]$, which generalize the notion of factorially closed subalgebras.

Recall that, for R-algebras $A \subset B$, A is **factorially closed**, or **fc**, in B if for $a, b \in B \setminus \{0\}$, the condition $ab \in A$ implies $a, b \in A$. By definition, if B is a UFD, then so is A. On the other hand, if $A^* \subsetneq B^*$, then A cannot be fc in B. With this observation, it is natural to generalize the notion of fc, indeed, the following concept has been introduced in the literature: A is **quasi-factorially closed**, or **qfc**, in B if for $a, b \in B \setminus \{0\}$, the condition $ab \in A$ implies $ua, vb \in A$ for some $u, v \in B^*$. Clearly, the implication "fc" $\Longrightarrow$ "qfc" holds.

Set $B := R[x_1^{\pm 1}, \ldots, x_n^{\pm 1}]$. We first establish a localization criterion for the qfc property. Using this criterion, we investigate monoid algebras $A = R[M]$ associated with submonoids $M \subset \mathbb{Z}^n$. We prove that $R[M]$ is qfc in B if and only if the group generated by M is a direct summand of $\mathbb{Z}^n$. As a consequence, when $n = 1$ and $M \subset \mathbb{N}$, the algebra $R[M]$ is qfc in B precisely when M is a numerical semigroup. Moreover, we clarify how the qfc condition is related to normality.

This is joint work with Shinya Kumashiro (Osaka Institute of Technology) and is available on arXiv:2603.25013.

New bounds on Castelnuovo-Mumford regularity of monomial curves and application to sumsets

Le Tuan Hoa

Institute of Mathematics, VAST

A monomial curve C is defined by a sequence of coprime integers $0 = a_0 < a_1 < \cdots < a_k =: d$. A gap of this sequence is $a_{i+1} - a_i - 1$. Gruson-Lazarsfeld-Peskine bound (1983) says that $\mathrm{reg}(C) \leq d - k + 2$, which is equal to the sum of all gaps plus 2. Lvovsky (1996) showed that it is enough to take the sum of two largest gaps plus 2.

In this paper, under some specific conditions, we give several new bounds which are better than Lvovsky's bound. Our method relies on the study of Apery sets and Frobenius numbers. From this we can give new criteria to check the (arithmetically) Cohen-Macaulay and Buchsbaum properties of C. Algorithms are provided to check these properties as well as to compute $\mathrm{reg}(C)$ and other invariants. We also give an application to studying the structure of sumsets.

This is a joint work with Doan Quang Tien.

On the depth of symbolic powers of squarefree monomial ideals

Yuji Muta

Okayama University

This talk is based on joint work with Naoki Terai (Okayama University). We will discuss our results on the Serre depth and the depth of unmixed squarefree monomial ideals. The Serre depth is an invariant introduced recently in relation to Serre's condition (S_r). We will explain basic properties of the Serre depth of unmixed Stanley-Reisner rings, as well as topics related to the depth non-increasing problem for symbolic powers and the classification of depth functions of symbolic powers for certain classes.

Ordered alternating paths and the depth of symbolic powers of cover ideals of graphs

Nguyen Thi Thanh Tam

Hung Vuong University

Let G be a simple graph with cover ideal $J(G)$ in a polynomial ring S in $|V(G)|$ variables. For a matching M of G, we denote by $\ell(M)$ the length of the longest M-alternating path in G. We define $\alpha_t(G)$ to be the maximum size of an ordered matching M of G such that $\ell(M) \leq 2t - 1$. We then prove that

$$\mathrm{depth}(S/J(G)^{(t)}) \leq |V(G)| - 1 - \alpha_t(G)$$

for all $t \geq 1$, where $J(G)^{(t)}$ denotes the t-th symbolic power of $J(G)$, and that equality holds when G is a forest.

This is joint work with Nguyen Thu Hang (TNUS) and Thanh Vu (IM–VAST).

Critical subgraphs and the regularity of symbolic powers of cover ideals of graphs

Nguyen Thu Hang

Thai Nguyen University of Sciences

Let G be a simple graph. We demonstrate a method for using t-admissible subgraphs of G to determine the regularity of the t-th symbolic power of the cover ideal of G.

For a graph G, denote by $\tau_{\max}(G)$ the maximum cardinality of a minimal vertex cover of G.

Theorem 1.1. Let G be a unicyclic graph whose unique cycle is C_n. Then

$$\operatorname{reg}(J(G)) = \begin{cases} \tau_{\max}(G)+1, & \text{if } n \equiv 1 \pmod 3 \text{ and } G \text{ is reducible to } C_n, \\ \tau_{\max}(G), & \text{otherwise.} \end{cases}$$

For powers of cover ideals of unicyclic graphs, we have:

Theorem 1.2. Let G be a unicyclic graph whose unique cycle is C_n. Assume that $\delta(G) = \tau_{\max}(G)$. Then for all $t \geq 2$,

$$\operatorname{reg}(J(G)^{(t)}) = \begin{cases} t\tau_{\max}(G)+1, & \text{if } n = 4 \text{ and } G \text{ is reducible to } C_4, \\ t\tau_{\max}(G), & \text{otherwise.} \end{cases}$$

This is a joint work with Thanh Vu.

References.

1. L. X. Dung, T. T. Hien, N. D. Hop, T. N. Trung, *Regularity and Koszul property of symbolic powers of monomial ideals*, Math. Z. 298 (2021), 1487–1522.
2. T. D. Dung, N. T. Hang, T. Vu, *Admissible subgraphs and the depth of symbolic powers of cover ideals of graphs*, arXiv:2605.03369v1.
3. N. C. Minh, L. D. Nam, T. D. Phong, P. T. Thuy, and T. Vu, *Comparison between regularity of small powers of symbolic powers and ordinary powers of an edge ideal*, J. Combin. Theory Ser. A 190 (2022), 105621.

Wednesday, September 9

Annihilators of local cohomology modules over modular invariant rings and Dickson polynomials

Tony J. Puthenpurakal

IIT Bombay

Let $\mathbb{F}_q$ be a finite field with $q = p^s$ elements. Let V be a d-dimensional vector space over $\mathbb{F}_q$ and let G be a subgroup of $\mathrm{GL}(V)$. Let $R = \mathbb{F}_q[V] = \mathrm{Sym}_{\mathbb{F}_q}(V^*)$ and let G act naturally on R. Set $S = R^G$. Let $d_{d,0}, d_{d,1}, \ldots, d_{d,d-1} \in S$ be the Dickson polynomials with $\deg d_{d,i} = q^d - q^i$. Let I be a homogeneous ideal of S and let $H^i_I(S)$ be the ith local cohomology module of S with respect to I. Let $J_i = \sqrt{\mathrm{ann}\, H^i_I(S)}$. Assume $J_i \neq 0$ and $\dim S/J_i = d-g$. Then we show that $d_{d,0}, \ldots, d_{d,g-1} \in J_i$. We give several applications of our results. An application is a considerably simpler proof of the Landweber–Stong conjecture. We also give a necessary and sufficient condition for S to have Serre's S_r-property in terms of Dickson polynomials.

Characterizations of regular local rings via analogues of $(*)$-properties

Shinnosuke Kosaka

Nagoya University

Let R be a commutative Noetherian local ring with residue field k. Characterizing the regularity of R in terms of syzygies of k has been frequently studied. A classical result due to Auslander, Buchsbaum, and Serre characterizes the regularity of R by the finiteness of the projective dimension of k.

Ghosh, Gupta, and Puthenpurakal [1] introduced the notion of $(*)$-properties. Typical examples of $(*)$-properties are being a semidualizing module and being a nonzero maximal Cohen–Macaulay module of finite injective dimension. They also proved that R is regular if there exists a surjective R-homomorphism from a finite direct sum of syzygies of k onto a finitely generated R-module satisfying a $(*)$-property. This result generalizes the theorem of Auslander, Buchsbaum, and Serre.

In this talk, we introduce analogues of $(*)$-properties and generalize the result of Ghosh, Gupta, and Puthenpurakal. As applications, we derive several criteria for the regularity of R.

References.

1. D. Ghosh, A. Gupta, T. J. Puthenpurakal, *Characterizations of regular local rings via syzygy modules of the residue field*, J. Commut. Algebra 10 (2018), no. 3, 327–337.
2. S. Kosaka, *Characterizing regular local rings via analogues of $(*)$-properties*, Comm. Algebra, published online (2026).

E-depth, Ef-depth and non-sequentially Cohen-Macaulay locus under small perturbations

Nguyen Xuan Linh

Hanoi University of Civil Engineering

We present recent joint works with D. T. Cuong and L. T. Nhan [1,2]. Let $(R, \mathfrak{m})$ be a Noetherian local ring, M a d-dimensional finitely generated R-module. It is well-known that M is Cohen–Macaulay if and only if there exists a system of parameters of M that is an M-sequence. If R is a quotient of a Cohen–Macaulay ring then M is generalized Cohen–Macaulay if and only if each system of parameters is an f-sequence. The class of sequentially Cohen–Macaulay modules is an important extension of the class of Cohen–Macaulay modules, introduced by R. Stanley (1996) and P. Schenzel (1998), and mainly devoted to studying non-unmixed modules. In a natural way, the class of sequentially generalized Cohen–Macaulay modules was defined by N. T. Cuong and L. T. Nhan in 2003.

For an M-regular element x, if M is sequentially Cohen–Macaulay then so is M/xM; however, in general the sequential Cohen–Macaulayness does not lift from M/xM to M. Therefore, the notions of sequential sequence and sequential f-sequence were introduced in [2] with the purpose of establishing an analogue for sequentially Cohen–Macaulay modules and sequentially generalized Cohen–Macaulay modules of the well-known characterizations of Cohen–Macaulay modules and generalized Cohen–Macaulay modules, where regular sequences and filter regular sequences are respectively replaced by sequential sequences and sequential f-sequences. In particular, M is sequentially Cohen–Macaulay if and only if M admits a system of parameters that is a sequential sequence.

Let $I = (x_1, \ldots, x_r)$ be an ideal of R. If $\varepsilon_1, \ldots, \varepsilon_r \in \mathfrak{m}^N$ for $N \gg 0$, then $I(\varepsilon) = (x_1 + \varepsilon_1, \ldots, x_r + \varepsilon_r)$ is called a small perturbation of the ideal I. In this talk, we define E-depth and Ef-depth of M in I and prove that E-depth (Ef-depth under mild conditions) of M in I is the common length of all maximal sequential sequences (sequential f-sequences) of M in I. We show that E-depth and Ef-depth do not decrease under small perturbations. We describe the non-sequentially Cohen–Macaulay locus of M in terms of support and the non-Cohen–Macaulay locus of deficiency modules. Then we prove that if $x_1, \ldots, x_r$ is a sequential f-sequence, then there exists an integer $N > 0$ such that for all $\varepsilon_1, \ldots, \varepsilon_r \in \mathfrak{m}^N$ we have

$$\dim \mathrm{nSCM}\big(M/(x_1 + \varepsilon_1, \ldots, x_r + \varepsilon_r)M\big) \leq \dim \mathrm{nSCM}\big(M/(x_1, \ldots, x_r)M\big).$$

References.

1. D. T. Cuong, L. T. Nhan, N. X. Linh, *On the perturbations of E-depth, Ef-depth and sequentially Cohen–Macaulay locus*, Vietnam J. Math. (2026), to appear.

2. N. X. Linh, L. T. Nhan, *On sequentially Cohen–Macaulay modules and sequentially generalized Cohen–Macaulay modules*, J. Algebra 678 (2025), 635–653.

Higher-dimensional Teter rings via the canonical trace

Taiga Ozaki

Institute of Science Tokyo

This talk is based on joint work with Sora Miyashita of the University of Osaka.

Teter rings are a classical class of Artinian Cohen-Macaulay rings which was introduced by W. Teter and tightly linked to Gorenstein properties. Beyond the Artinian case, the higher–dimensional local analogue was formulated by T. J. Puthenpurakal. In our research, we introduce a graded analogue that unifies the Artinian and local viewpoints.

In this talk, I will first review the definition and introduce some properties of Teter rings in the (semi-)standard graded case. After that, I will present a characterization of the Teterness of numerical semigroup rings, which serve as a classical class of non-standard one-dimensional Cohen-Macaulay graded rings.

Cohen-Macaulayness of formal fibers and dimension of local cohomology modules

Tran Do Minh Chau

Thai Nguyen University of Education

Let $(R, \mathfrak{m})$ be a Noetherian local ring, M a finitely generated R-module of dimension d. Set

$$\mathfrak{a}(M) := \mathfrak{a}_0(M) \cdots \mathfrak{a}_{d-1}(M),$$

where $\mathfrak{a}_i(M) := \operatorname{Ann}_R H^i_{\mathfrak{m}}(M)$ for $i \geq 0$. In this talk, we study the Cohen–Macaulayness of formal fibers of R in relation with the dimension $\dim(R/\mathfrak{a}(M))$.

We prove that $\dim(R/\mathfrak{a}(M)) < d$ if and only if $R/\mathfrak{p}$ is unmixed and the generic formal fiber of $R/\mathfrak{p}$ is Cohen–Macaulay for all $\mathfrak{p} \in \operatorname{Supp}_R(M)$ with $\dim(R/\mathfrak{p}) = d$. In general, $R/\mathfrak{p}$ is unmixed and the generic formal fiber of $R/\mathfrak{p}$ is Cohen–Macaulay for all $\mathfrak{p} \in \operatorname{Supp}_R(M)$ with $\dim(R/\mathfrak{p}) > \dim(R/\mathfrak{a}(M))$. As applications, we explore the structure of local rings and the dimension and closedness of the non-Cohen–Macaulay locus of finitely generated modules.

On the Frobenius test exponents

Pham Hung Quy

FPT University

In this talk, I will present our recent results on bounds for the Frobenius test exponent of a local ring of positive characteristic.

- We prove that the Frobenius test exponent is always greater than or equal to the Hartshorne–Speiser–Lyubez number of the ring.
- If the ring is generalized Cohen–Macaulay, we establish a sharp upper bound for the Frobenius test exponent.

Extended degrees and sequentially generalized Cohen-Macaulay modules

Pham Hong Nam

TNU-University of Sciences

We characterize the sequential generalized Cohen–Macaulayness of modules in two ways: (1) in terms of the relation between the unmixed degrees and the arithmetic degrees; (2) in terms of the tight connection among unmixed torsions, arithmetic degrees, and Hilbert coefficients. As applications, we provide characterizations of Cohen–Macaulay modules, generalized Cohen–Macaulay modules, and sequentially Cohen–Macaulay modules.

On structure of certain classes of modules over commutative Noetherian rings

Le Thanh Nhan

Institute of Mathematics, VAST

In this presentation, I will review some research achievements of Prof. Nguyen Tu Cuong concerning the structure of modules over commutative Noetherian rings. Additionally, the talk will highlight representative results from his students, which inherit and further develop his research directions as well as the fundamental concepts he introduced. Specifically, we will focus on the following main topics:

1. Generalized Cohen-Macaulay modules and filter-regular sequences;

2. Non-Cohen-Macaulay singularities: p-standard systems of parameters, polynomial type, and structure of finitely generated modules over local rings;

3. Local homology and the structure of linearly compact modules, Artinian modules;

4. Local cohomology, splitting properties, finiteness properties, and asymptotic stability.

Thursday, September 10

Local rings with zero-dimensional formal fibers and annihilators of Artinian modules

Tran Nguyen An

Thai Nguyen University of Education

This is a joint work with Nguyen Thi Anh Hang (Thai Nguyen University of Education) and Le Thanh Nhan (Institute of Mathematics, VAST). This paper deals with the classification of commutative Noetherian local rings with zero-dimensional formal fibers. We extend the concept of AB-5* local ring to the notions of radically AB-5* local ring and bijective spectrum local ring. All such local rings have zero-dimensional formal fibers and the canonical injection into the completion satisfies the going-up property. Among the results, we characterize the properties of being AB-5*, radically AB-5*, having a bijective spectrum for local rings in terms of the compatibility of the annihilators of Artinian modules under completion as well as the relationships between the ideals of a local ring and the ideals of its completion. Some examples are given to clarify the results.

Reference.

1. N. T. A. Hang, T. N. An and L. T. Nhan, *Local rings with zero-dimensional formal fibers and annihilators of Artinian modules*, submitted.

A q-analogue of dualizing complexes

Yuto Yamada

Institute of Science Tokyo

Roughly speaking, a q-analogue is a method that recovers a target theory in the limit as $q \to 1$. Classically, such ideas appeared in analytic contexts, for example in the theory of q-differentials. In recent years, algebraic objects such as the q-de Rham complex have also been developed. More recently, Garoufalidis-Scholze-Wheeler-Zagier and Wagner proposed a new approach to q-analogues in which the Habiro ring plays a central role. This theory is also closely related to the notion of q-Witt vectors. Since dualizing complexes provide a natural framework for formulating duality phenomena in algebraic geometry, especially in singular setting, one motivation for this study is to ask whether they admit a reasonable q-deformation. The Habiro-theoretic viewpoint suggests that such a deformation could be assembled from its finite-level, q-Witt-theoretic approximations. In this talk, following this perspective, I will first define the notion of q-Witt dualizing complexes and explain some of their properties using q-Witt theory . Then, I will define a q-analogue of dualizing complexes by "gluing" these q-Witt dualizing complexes. Finally, I would like to discuss several pieces of evidence suggesting that this construction indeed deserves to be regarded as a genuine q-analogue.

Regular rings and perfectoid towers

Kazuki Hayashi

Institute of Science Tokyo

Kunz's theorem characterizes regularity of Noetherian rings of positive characteristic via the flatness of the Frobenius map. This result also reveals a close relationship between regular rings of positive characteristic and perfect rings. B. Bhatt, S. B. Iyengar, and L. Ma extended this perspective to mixed characteristic, with perfectoid rings playing the role of perfect rings. On the other hand, S. Ishiro, K. Nakazato, and K. Shimomoto introduced perfectoid towers as Noetherian approximations of perfectoid rings. In this talk, we discuss the relationship between regular rings of mixed characteristic and perfectoid towers. In particular, we present a Kunz-type regularity criterion formulated in terms of perfectoid towers.

Local and global aspects of perfectoid pure singularities

Ryo Ishizuka

Institute of Science Tokyo

Bhatt, Iyengar, and Ma characterized regular local rings in mixed characteristic via faithfully flat maps to perfectoid rings. Recently, perfectoid pure singularities were introduced as an analogue of F-pure singularities, which correspond well to singularities in characteristic $p \geq 0$. In this talk, I will introduce their global variant, (lim-)perfectoid splitting. I will discuss its properties and examples, focusing on the correspondence between the local and global theories. This is joint work with Shou Yoshikawa.

Towards a classification of integrally closed modules over two-dimensional regular local rings

Futoshi Hayasaka

Okayama University

The theory of integrally closed ideals in two-dimensional regular local rings was developed by Zariski. One of its fundamental structural results is the product theorem, which asserts that the product of any two integrally closed ideals is again integrally closed. Kodiyalam generalized this result to integrally closed modules over two-dimensional regular local rings.

In recent work, we characterized the ideals in two-dimensional regular local rings that arise as the ideals of maximal minors of indecomposable integrally closed modules, particularly in the case where the rank is at most three. As a consequence, we obtained a large class of indecomposable integrally closed modules.

In this talk, we classify the isomorphism classes of integrally closed modules of rank 2 whose ideals of maximal minors have order 2 over a two-dimensional regular local ring. As a consequence, we show that, for a given integrally closed ideal of order 2, there exist only finitely many isomorphism classes of integrally closed modules of rank 2 with the given ideal.

This is joint work in progress with Vijay Kodiyalam.

The weak Lefschetz property for tensor products of Artinian monomial algebras and its applications to lollipop graphs

Nguyen Duy Phuoc

University of Education, Hue University

This is joint work with Quang Hoa Tran and Nguyen Thanh Son Tran. In this talk, we provide a condition under which tensor products of Artinian monomial algebras and complete quadratic monomial algebras have the weak Lefschetz property. As an application, we classify the weak Lefschetz property of the Artinian algebras $A(L_{m,n})$, which are defined by the edge ideals of the lollipop graphs $L_{m,n}$ together with the squares of the variables.

Classification on nearly Gorenstein rational surface singularities with multiplicity 4

Kyosuke Maeda

Nihon University

It is well known that Gorenstein rational singularities are rational double points. The classification of nearly Gorenstein rational triple points was given in joint work with Ken-ichi Yoshida. Furthermore, a criterion for nearly Gorenstein rational singularities and the classification of nearly Gorenstein rational singularities with an almost reduced fundamental cycle were established in joint work with Tomohiro Okuma, Kei-ichi Watanabe, and Ken-ichi Yoshida. In this talk, based on the results of the latter joint work by the four authors, we present a classification of nearly Gorenstein rational surface singularities with multiplicity 4.

Weak del Pezzo surfaces are characterized by the existence of 2-tilting bundles

Ryu Tomonaga

The University of Tokyo

Tilting bundles provide a fundamental bridge between algebraic geometry and representation theory. For a tilting bundle on a smooth proper d-dimensional variety, the global dimension of its endomorphism algebra is at least d, and the most significant case is when this lower bound is attained. Such a tilting bundle, called a d-tilting bundle, fits into the framework of the derived McKay correspondence and higher Auslander–Reiten theory.

In this talk, we present the following results.

(1) If a d-dimensional smooth proper variety admits a d-tilting bundle, then its anti-canonical bundle is semiample and big. In particular, the variety is weak Fano.

(2) Every weak del Pezzo surface admits a 2-tilting bundle.

Together, these results give an affirmative answer to a conjecture of Daniel Chan for the variety case: a smooth projective surface admits a 2-tilting bundle if and only if it is a weak del Pezzo surface.

As an application of (2), we show the following result.

(3) Every Du Val del Pezzo cone admits a non-commutative crepant resolution.

This extends a result of Van den Bergh for del Pezzo cones.

Friday, September 11

Absolutely Koszul edge ideals and matching-cochordal decomposable graphs

Nguyen Dang Hop

Institute of Mathematics, VAST

A standard graded k-algebra R is absolutely Koszul if every finitely generated graded R-module has a componentwise linear syzygy module. The class of absolutely Koszul algebras subsumes all complete intersection of quadrics, and all algebras whose defining ideals have 2-linear resolutions (e.g. coordinate rings of varieties of minimal degree over the complex numbers). We discuss the classification of absolutely Koszul algebras defined by monomial ideals. It is not hard to show that we can restrict to quadratic squarefree monomial ideals, namely edge ideals of graphs. We classify completely graphs on at most 6 vertices whose edge ideals define absolutely Koszul rings. We also classify completely paths, cycles, and forests with absolutely Koszul edge ideals. We discuss the closely related problem of classifying the so-called matching-cochordal decomposable graphs.

An almost p-standard system of parameters and the sequential Cohen-Macaulayness of idealizations

Phan Van Loc

Hanoi Pedagogical University 2

Let $(R, \mathfrak{m})$ be a Noetherian local ring of dimension r and M be a finitely generated R-module of dimension d. In this talk, we characterize the sequential Cohen-Macaulayness of an idealization $R \ltimes M$ via the length function and Hilbert coefficients of the idealization $R \ltimes M$ with respect to an almost p-standard system of parameters of the form $(x_1, 0), \ldots, (x_r, 0)$.

This is from joint work with Assoc. Prof. Dr. Pham Hong Nam.

Almost p-standard systems of parameters and the sequential generalized Cohen-Macaulayness of idealizations

Tran Thi Van Anh

Hanoi Pedagogical University 2

Let $(R, \mathfrak{m})$ be a Noetherian local ring of dimension r and M be a finitely generated R-module. We characterize the sequential generalized Cohen–Macaulayness of the idealization $R \ltimes M$ via the length function and Hilbert coefficients of $R \ltimes M$ relative to an almost p-standard system of parameters of the form $(x_1, 0), \ldots, (x_r, 0)$ (a very strict subclass of d-sequences). As an application, we characterize the Cohen–Macaulayness of the base ring and the module in terms of the vanishing of the Chern number of the idealization.

This is joint work with N. V. Dinh, N. T. Hang and P. H. Nam.

Trace ideals of exterior powers of the module of differentials

Sora Miyashita

The University of Osaka

For each $i \geq 0$, we study the trace ideal of the i-th exterior power of the module of differentials. We show that these ideals characterize the polynomial rank of graded rings and the formal power series rank of complete local rings, namely the maximal number of variables for a polynomial or formal power series extension over a subring. For the top exterior power, we introduce the differential trace and prove that it precisely defines the singular locus of reduced equidimensional local or graded rings. Motivated by this, we introduce and investigate nearly regular rings, which are certain Noetherian rings whose top differential trace contains the maximal ideal.

A generalization of maximal Cohen-Macaulay approximation theorem and its application

Hiroki Matsui

Hiroshima University

In this talk, we generalize the maximal Cohen-Macaulay approximation theorem to the non-Cohen-Macaulay setting. The key ingredient is to use maximal Cohen-Macaulay complexes instead of maximal Cohen-Macaulay modules. Also, as an application, we study the relationship between the Auslander condition and its dual condition, the Huneke-Jorgensen condition.

Toric rings of signed posets

Koji Matsushita

The University of Tokyo

In this talk, we consider toric rings associated with signed posets, which are combinatorial objects arising from root systems. These rings can be regarded as a generalization of Hibi rings. We discuss our results on the divisor class groups of these rings and on a characterization of when they are Gorenstein. If time permits, we will also explain a characterization of their conic divisorial ideals.

Pseudo-Frobenius numbers and defining ideals in stretched numerical semigroup rings

Do Van Kien

Hanoi Pedagogical University 2

This is based on joint works with Naoyuki Matsuoka and Taiga Ozaki.

Let $H = \langle a_1, a_2, \ldots, a_n \rangle$ be a numerical semigroup and

$$\mathrm{PF}(H) = \{x \in \mathbb{Z} \setminus H \mid x + h \in H \text{ for all } 0 \neq h \in H\}$$

the set of pseudo-Frobenius numbers of H. Let $k[H] = k[t^{a_1}, t^{a_2}, \ldots, t^{a_n}]$ be the numerical semigroup ring of H over a field k and I_H the defining ideal of $k[H]$.

The pseudo-Frobenius numbers of a numerical semigroup H are closely related to the structure of the defining ideal of its semigroup ring $k[H]$. In this talk, we prove a conjecture of Cuong, Kien, Matsuoka,

and Truong by establishing a criterion for the defining ideal I_H to be generated by the 2×2 minors of a $2 \times n$ matrix in terms of the pseudo-Frobenius numbers, assuming that $k[H]/(t^a)$ is stretched, where a is the multiplicity of H. We also give numerical conditions ensuring that the tangent cone of $k[H]$ is Cohen–Macaulay.

Admissible subgraphs and the depth of symbolic powers of cover ideals of graphs

Tran Duc Dung

Thai Nguyen University of Sciences

Let G be a simple graph. We introduce the notion of t-admissible subgraphs of G and show how to use them to compute the depth of the t-th symbolic powers of the cover ideal of G. As an application, we prove that

$$\operatorname{depth}\big(S/J(C_n)^{(t)}\big) = n - 1 - \left\lfloor \frac{tn}{2t+1} \right\rfloor$$

for all $t \geq 2$ and $n \geq 3$, where $S = K[x_1, \ldots, x_n]$ and $J(C_n)$ is the cover ideal of the cycle on n vertices. This is joint work with Nguyen Thu Hang and Thanh Vu.

Invariant chains of graphs

Le Van Dinh

FPT University

Invariant chains of graphs provide a natural combinatorial analogue of invariant chains of ideals arising in the theory of equivariant Noetherianity. Such chains consist of graphs whose vertex sets grow with the ambient set $[n]$, while their edge sets are compatible with the action of the monoid of strictly increasing maps on the positive integers. In this talk, I will discuss the asymptotic behavior of several combinatorial invariants and structures associated with these chains, including independence complexes, clique complexes, chromatic numbers, matching numbers, and minimal and admissible paths. This is joint work with D. T. Hoang and M. Koley.

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Useful Information

Venue

Talks will be held in **Meeting Room No. 3** on the main campus of the University of Khanh Hoa, 01 Nguyen Chanh, Nha Trang, Khanh Hoa. The room is located inside the university campus; directions to the meeting room are provided on the next page.

Facilities and Wi-Fi

Meeting Room No. 3 is equipped with a projector and a whiteboard (to be used with whiteboard markers). Wi-Fi will be available in the conference room. Access information will be provided during the seminar.

Banquet

The conference banquet will be held on **Wednesday evening, September 9**. The location will be announced later.

How to get to the University of Khanh Hoa?

The main campus of the University of Khanh Hoa is located in central Nha Trang at:

University of Khanh Hoa
01 Nguyen Chanh, Nha Trang, Khanh Hoa, Vietnam

Participants staying at the conference hotel will be at:

Yasaka Saigon Nha Trang Hotel & Spa
(Yasaka Nha Trang Hotel)
18 Tran Phu Street, Nha Trang, Khanh Hoa, Vietnam

The university is within easy walking distance of the hotel. From the hotel, walk south along Tran Phu Street, turn right onto Nguyen Chanh Street, and continue to No. 01. The walk is approximately 450–500 metres and normally takes about 6–8 minutes.

Conference Hall Location Map

The marked route leads from the main gate on Nguyen Chanh Street to the conference hall.

Previous Japan–Vietnam Joint Seminars on Commutative Algebra

No.	Dates	Location	Country
1	August 2001	Yokohama	Japan
2	March 20–25, 2006	Ikuta	Japan
3	December 3–7, 2007	Hanoi	Vietnam
4	February 17–21, 2009	Ikuta	Japan
5	January 5–9, 2010	Hanoi	Vietnam
6	December 11–16, 2010	Hayama	Japan
7	December 12–16, 2011	Quy Nhon	Vietnam
8	March 21–25, 2016	Ha Long	Vietnam
9	November 18–22, 2016	Hayama	Japan
10	September 10–14, 2018	Hue	Vietnam
11	March 28–30, 2023	Hanoi	Vietnam

By and for young mathematicians