CHƯƠNG TRÌNH HỘI THẢO

MỘT SỐ VẤN ĐỀ ĐƯƠNG ĐẠI CỦA TOÁN HỌC
TIỂU BAN ĐẠI SỐ VÀ LÝ THUYẾT SỐ

Ngày 24/12/2025, Viện Toán học

Địa điểm: Phòng 612, nhà A6, Viện Toán học, 18B Hoàng Quốc Việt, Hà Nội

Thứ Tư, ngày 24/12/2025

Buổi sáng

Chủ tọa: Trần Giang Nam

08:00 – 08:30	Đăng ký đại biểu
08:30 – 09:10	GS. TSKH. Ngô Việt Trung (Viện Toán học) *Local cohomology at the beginning*
09:10 – 09:50	PGS.TS. Nguyễn Duy Tân (Trường Đại học Bách Khoa Hà Nội) *On divisibility relation graphs*
09:50 – 10:10	Tiệc trà
10:10 – 10:50	PGS. TS. Đoàn Trung Cường (Viện Toán học) *Direct summands of the syzygies of the residue field of a local ring*
10:50 – 11:30	GS. TS. Lê Thị Thanh Nhàn (Viện Toán học) *E-depth, EF-depth and non sequentially Cohen-Macaulay locus under small pertubations*
11:30 – 13:30	Ăn trưa

Buổi chiều

Chủ tọa: Nguyễn Đăng Hợp

14:00 – 14:40	GS. TS. Nguyễn Quốc Thắng (Viện Toán học) *Remarks on dimension inequality in descending and ascending series of solvable groups*
14:40 – 15:20	TS. Ngô Trung Hiếu (Viện Toán học) *Central values of degree one L-functions*
15:20 – 15:40	Tiệc trà
15:40 – 16:20	PGS. TS. Phạm Hùng Quý (Trường Đại học FPT) *Buchsbaum property in positive characteristic*
16:20 – 17:00	PGS. TS. Trần Nam Trung (Viện Toán học) *Cohen-Macaulayness of powers of edge ideals of weighted oriented graphs*
17:00	Ăn tối ở Nhà hàng Soulmate.

TÓM TẮT BÁO CÁO

Direct summands of the syzygies of the residue field of a local ring

Đoàn Trung Cường (Viện Toán học)

Let (R, m, K) be a Noetherian local ring. It is well known that the sequence of Betti numbers of the residue field K has extremal growth among all finitely generated R-modules. On the other hand, much less is known about the structure of the syzygy modules of K. In this talk, I will present some recent results on the existence of special direct summands in certain syzygies of K and discuss their connections with the structure of the ring R.

Central values of degree one L-functions

Ngô Trung Hiếu (Viện Toán học)

A basic quest in number theory is to understand the distribution of values of L-functions inside the critical strip, specifically the central values on the line of symmetry. In this talk we will address $GL(1)$ L-functions (the Riemann zeta function and Dirichlet L-functions) and give an overview of the challenging problems of computing moments of their central values. We will discuss Chowla's conjecture concerning nonvanishing of central values of Dirichlet L=functions and recent progress using the mollifier approach. This is based on joint work with Rizwanur Khan and Djordje Milicevic, and with Hung Bui and Alexandra Florea.

E-depth, EF-depth and non sequentially Cohen-Macaulay locus under small pertubations

Lê Thị Thanh Nhàn (Viện Toán học)

This is a joint work with D.T. Cuong and N.X. Linh [1].
Let $(R, \mathfrak{m})$ be a Noetherian local ring, M a finitely generated R-module, I an ideal of R. The classes of sequentially Cohen-Macaulay modules and sequentially generalized Cohen-Macaulay modules are important extensions of the classes of Cohen-Macaulay modules and generalized Cohen-Macaulay modules respectively, see [6], [5], [2]. Note that the property of being a sequentially Cohen-Macaulay module on M is inherited by M/xM if x is a regular element of M. However, the sequential Cohen-Macaulayness does not lift from M/xM to M for a regular element x in general. Because of this inadequacy, several other kinds of sequences called sequential sequence and sequential f-sequence was introduced [3] that can be used for establishing an analogue for the sequentially Cohen-Macaulay modules and sequentially generalized Cohen-Macaulay modules of the well-known characterizations of Cohen-Macaulay modules and generalized Cohen Macaulay modules,

where the roles of regular sequences and filter regular sequences are respectively replaced by that of sequential sequences and sequential f-sequences.
On the other hand, for an ideal I of R, the question of which properties and numerical invariants related to M and I are preserved under small perturbations has attracted the interest among mathematicians. The behavour of Hilbert-Samuel function, dimension of non Cohen-Macaulay locus under small pertubations are investigated, see [4].
In this talk, we define and study the $-depth_R(I, M)$ and $-depth_R(I, M)$. We describe the non sequentially Cohen-Macaulay locus (M). Under small pertubations, we show that the E-depth and Ef-depth do not decrease, and that the dimension of non sequentially Cohen-Macaulay locus with respect to a sequential f-sequence does not increase.

Tài liệu

[1] D. T. Cuong, L. T. Nhan, N. X. Linh, E-depth, Ef-depth and non sequentially Cohen-Macaulay locus under small pertubations, Preprint.

[2] N. T. Cuong and L. T. Nhan, Pseudo Cohen-Macaulay and pseudo generalized Cohen-Macaulay modules, *J. Algebra,* **267** (2003), 156-177.

[3] N. X. Linh, L. T. Nhan, On sequentially Cohen-Macaulay modules and sequentially generalized Cohen-Macaulay modules, *J. Algebra,* **678** (2025) 635-653.

[4] L. Ma, P. H. Quy and I. Smirnov, Filter regular sequence under small perturbations, *Math. Ann.*, **378** (2020), 243-254.

[5] P. Schenzel, On the dimension filtration and Cohen-Macaulay filtered modules, Commutative algebra and algebraic geometry (Ferrara), 245-264, Lecture Notes in Pure and Appl. Math., 206, Dekker, New York, 1999.

[6] R. P. Stanley, "Combinatorics and Commutative Algebra", Second edition, Birkhäuser Boston, 1996.

On divisibility relation graphs

Nguyễn Duy Tân (Trường Đại học Bách Khoa Hà Nội)

For each positive integer n, the divisibility relation graph D_n is defined as the graph whose vertices are the positive divisors of n, with an edge between two distinct vertices whenever one divides the other. In this talk, we discuss the spectral properties of these graphs (joint work with Jonathan L. Merzel, Jan Minac and Tung T. Nguyen)

Remarks on dimension inequality in descending and ascending series of solvable groups

Nguyễn Quốc Thắng (Viện Toán học)

For connected nilpotent or solvable algebraic groups, there were some well known estimations on dimension of subgroups in the descending derived series and ascending central series for connected solvable or nilpotent algebraic groups, which are very useful when one uses mathematical induction to investigate the structure of the given linear algebraic groups. Our aim in this talk is to investigate to what extent one can extend these estimates to the case of non-connected linear algebraic solvable or nilpotent groups.

Cohen-Macaulayness of Powers of Edge Ideals of Weighted Oriented Graphs

Trần Nam Trung (Viện Toán học)

An oriented graph $D = (V(D), E(D))$ consists of a simple underlying graph G in which each edge is oriented, i.e., it is a directed graph with no multiple edges or loops. The elements of $E(D)$ are denoted by ordered pairs to reflect the orientation. For example, (u, v) represents an edge directed from u to v. A *vertex-weighted* (or simply weighted) oriented graph D is a graph equipped with a weight function $\omega\colon V(D) \to \mathbb{Z}_{>0}$. The pair (D, ω) is called a weighted oriented graph. When there is no confusion, we will simply use D to represent this pair.
Let $R = K[x_1, \ldots, x_n]$ be a polynomial ring with n variables over a field K. Assume that $V(D) = \{1, 2, \ldots, n\}$. The edge ideal of D is defined as

$$I(D) = (x_i x_j^{\omega(j)} \mid (i, j) \in E(D)).$$

We will prove that its symbolic powers $I(D)^{(t)}$ are Cohen-Macaulay for all $t \geqslant 1$ if and only if the underlying graph G is composed of a disjoint union of some complete graphs. We also completely characterize the Cohen-Macaulayness of the ordinary powers $I(D)^t$ for all $t \geqslant 2$. Furthermore, we provide a criterion for determining whether $I(D)^t = I(D)^{(t)}$. This is a joint work with Truong Thi Hien, Jiaxin Li and Guangjun Zhu.

Local cohomology at the beginning

Ngô Việt Trung (Viện Toán học)

The talk is about the origin and some applications of local cohomology in Commutative Algebra.

Buchsbaum property in positive characteristic

Phạm Hùng Quý (Trường Đại học FPT)

Buchsbaum rings have been extensively studied since the 1970s. This class is a natural generalization of Cohen–Macaulay rings. In this talk, we investigate classes of rings with the properties of tight Buchsbaum and Frobenius Buchsbaum in positive characteristic. The talk is based on joint work with Linquan Ma (2023) and with Kriti Goel, Kyle Maddox, Lance Edward Miller, and Austyn Simpson (preprint).