VIETNAM ACADEMY
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INSTITUTE OF MATHEMATICS

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SUMMARY OF Ph.D. THESIS

Title: Convergence Rates for the Tikhonov Regularization of Coefficient Identification Problems in Elliptic Equations

Speciality: Differential and Integral Equations.

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In this thesis we investigate convergence rates for Tikhonov regularization of the problem of identifying

1) the coefficient $q \in L^\infty(\Omega)$ in the Neumann problem

$$-\mathrm{div}(q\nabla u) = f \text{ in } \Omega \subset \mathbb{R}^d, d \geq 1$$
$$q\frac{\partial u}{\partial n} = g \text{ on } \partial\Omega$$

2) the coefficient $a \in L^\infty(\Omega)$ in the Neumann problem

$$-\Delta u + au = f \text{ in } \Omega,$$
$$\frac{\partial u}{\partial n} = g \text{ on } \partial\Omega$$

when u is imprecisely given by $z^\delta \in H^1(\Omega)$ with $\|u - z^\delta\|_{H^1(\Omega)} \leq \delta$, $\delta > 0$.

We use the convex functional

$$q \to \frac{1}{2}\int_\Omega q|\nabla(U(q) - z^\delta)|^2 dx, \;\; q \in Q_{ad}$$

for identifying the coefficient q and the convex functional

$$a \to \frac{1}{2}\int_\Omega |\nabla(U(a) - z^\delta)|^2 dx + \frac{1}{2}\int_\Omega a(U(a) - z^\delta)^2 dx, \;\; a \in A_{ad}$$

for identifying the coefficient a. Here, $U(q)$ and $U(a)$ are the coefficient-to-solution maps for the first and second Neumann problems with Q_{ad} and A_{ad} being the admissible sets, respectively. Since coefficient identification problems are ill-posed, we apply Tikhonov regularization to minimizing these convex functionals over admissible sets and establish the convergence rates for this approach.

New results presented in the thesis

1. Convergence rates for Tikhonov regularized solutions of coefficient identification problems with the penalty fuctional being $\|\cdot\|^2_{L^2(\Omega)}$ in the $L^2(\Omega)$-norm.

2. Convergence rates for Tikhonov regularized solutions of coefficient identification problems with the penalty fuctional being total variation $\int_\Omega |\nabla(\cdot)|$ in the sense of the Bregman distance.

3. Convergence rates for Tikhonov regularized solutions of coefficient identification problems with the penalty fuctional being $\frac{1}{2}\|\cdot\|^2_{L^2(\Omega)} + \int_\Omega |\nabla(\cdot)|$ in the $L^2(\Omega)$-norm and in the Bregman distance.

Hanoi, 16 July, 2012
Ph.D. Student

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