

SUMMARY OF PHD DISSERTATION

Dissertation Title: **Some Parametric Optimization Problems in Mathematical Economics**

Speciality: Applied Mathematics

Speciality code: 9 46 01 12

PhD student: **Vu Thi Huong**

Supervisor: **Prof. Dr.Sc. Nguyen Dong Yen**

Training Institute: Institute of Mathematics, Vietnam Academy of Science and Technology

This thesis focuses on qualitative properties (solution existence, optimality conditions, stability, and sensitivity) of optimization problems in consumption economics, production economics, and optimal economic growth models. The thesis has two parts, which are divided into five chapters.

Part I, which includes the first two chapters, studies the stability (the continuity property, the Lipschitz property, the Lipschitz-like property, and the Lipschitz-Hölder property) and the differential stability (the Fréchet/limiting coderivatives, the Fréchet/limiting subdifferentials of the infimal nuisance function, upper and lower estimates for the upper and the lower Dini directional derivatives of the indirect utility function) of the consumer problem named *maximizing utility subject to consumer budget constraint with varying prices*. Mathematically, this is a parametric optimization problem; and it is worthy to stress that the problem considered here also presents the producer problem named *maximizing profit subject to producer budget constraint with varying input prices*. Both problems are basic ones in microeconomics.

Part II of the dissertation includes next three chapters. In Chapters 3 and 4, *a maximum principle for finite horizon optimal control problems with state constraints is analyzed via parametric examples*. Each of those examples is an optimal control problem with five parameters. The difference among those are in the appearance of state constraints: The first one does not contain state constraints, the second one is a problem with unilateral state constraints, and the third one is a problem with bilateral state constraints. The analysis in these chapters not only helps to understand advanced tools from optimal control theory (Filippov's existence theorems, the maximum principles) in depth, but also serves as a sample of applying them to meaningful prototypes of economic optimal growth models in macroeconomics. Chapter 5 establishes a series of theorems on solution existence for *optimal economic growth problems* in general forms as well as in some typical ones and synthesis of optimal processes for one of such typical problems.

The main results of the thesis include:

1) Results on the upper continuity, the lower continuity, and the continuity of the budget map, the indirect utility function, and the demand map; the Robinson stability and the Lipschitz-like property of the budget map; the Lipschitz property of the indirect utility function; the Lipschitz-Hölder property of the demand map.

2) Formulas for computing the Fréchet/limiting coderivatives of the budget map; the Fréchet/limiting subdifferentials of the infimal nuisance function, upper and lower estimates for the upper and the lower Dini directional derivatives of the indirect utility function.

3) The syntheses of finitely many processes suspected for being local minimizers for parametric optimal control problems without/with state constraints.

4) Three theorems on solution existence for optimal economic growth problems in general forms as well as in some typical ones, and the synthesis of optimal processes for one of such typical problems.

5) Interpretations of the economic meanings for most of the obtained results.

Supervisor

Hanoi, February 26, 2020

PhD student

Prof. Dr.Sc. Nguyen Dong Yen

Vu Thi Huong