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THE RELATION BETWEEN $U(1)$-COVERING AND TWO-FOLD COVERING IN REPRESENTATION THEORY OF LIE GROUPS

TRAN DAO DONG and TRAN VUI

Abstract. *To avoid the Mackey's obstructions when reducing Kirillov's orbit method to special contexts, M. Duflo lifted the character of the stabilizer to two-fold covering by using metaplectic structures. Our purpose is to suggest a lifting of the character to U(1)-covering via Mp^c-structures instead of metaplectic structures and then show that there exists a bijection relation between U(1)-admissible orbit and Z_2-admissible orbits.*

INTRODUCTION

Let G be a connected and simply connected Lie groups. In order to find irreducible unitary representations of G, Kirillov's orbit method furnishes a procedure of quantization starting from linear bundles over a G-homogeneous symplectic manifold (see [4]). In 1979-1980, Do Ngoc Diep [1, 2] has proposed the procedure of multidimensional quantization for general case, starting from arbitrary irreducible bundles. This procedure could be viewed as a geometric version of the construction of M. Duflo [3].

In this paper, we lift the character of the stabilizer to $U(1)$-covering via Mp^c-structures instead of metaplectic structures by using the technique of Robinson-Rawnsley [5]. In the Bargmann-Segal model, we firstly define and describe $U(1)$-admissible orbits and then verify the bijection relation between these orbits and Z_2-admissible orbits that proposed by M. Duflo and D. Vogan.

1. THE Mp^c-STRUCTURE AND $U(1)$-ADMISSIBLE ORBITS

Let G be a connected and simply connected Lie group. Denote by $\mathcal{G}$ the Lie algebra of G and $\mathcal{G}^*$ its dual space. The group G acts on $\mathcal{G}$ by the adjoint representation and on $\mathcal{G}^*$ by the coadjoint representation or simply K-representation. Denote by G_F the stabilizer of $F \in \mathcal{G}^*$ and by $\mathcal{G}_F$ its Lie algebra. Denote Ω the K-orbit in passing $F \in \mathcal{G}^*$. Let B_F be the bilinear form on $\mathcal{G}$ given by

$$B_F(X,Y) = \langle F, [X,Y]\rangle,\ \forall X, Y \in \mathcal{G}\,.$$

We see that $\mathrm{Ker} B_F = \mathcal{G}_F$. Write $\widetilde{B}_F$ for the symplectic form on $\mathcal{G}/\mathcal{G}_F$ induced from B_F and B_Ω the Kirillov 2-form on Ω (see [4, §15]).

1.1. The Mp^c-structure

Denote by $Sp(\mathcal{G}/\mathcal{G}_F,\ \widetilde{B}_F)$ or simply $Sp(\mathcal{G}/\mathcal{G}_F)$ the symplectic group of $\mathcal{G}/\mathcal{G}_F$ consists of all the real automorphisms that preserve the symplectic form $\widetilde{B}_F$.

If $\mathcal{G}_F \neq \mathcal{G}$, then (see [5, §2]) for every $f \in Sp(\mathcal{G}/\mathcal{G}_F)$ there exists a unitary operator U on the Bargmann space $\mathbb{H}(\mathcal{G}/\mathcal{G}_F)$ of $\mathcal{G}/\mathcal{G}_F$ such that

$$\widetilde{X} \in \mathcal{G}/\mathcal{G}_F \Rightarrow W(f\widetilde{X}) = UW(\widetilde{X})U^{-1},$$

where $W : \mathcal{G}/\mathcal{G}_F \to \mathrm{Aut}\,\mathbb{H}(\mathcal{G}/\mathcal{G}_F)$ is a projective irreducible unitary representation of the additive group of $\mathcal{G}/\mathcal{G}_F$ with multiplier $\exp\frac{1}{2i\hbar}\widetilde{B}_F$. We write $\sigma(U) = f$ when this holds.

Define $Mp^c(\mathcal{G}/\mathcal{G}_F, \widetilde{B}_F)$ or simply $Mp^c(\mathcal{G}/\mathcal{G}_F)$ the subgroup of $\mathrm{Aut}\,\mathbb{H}$ consists of all unitary operators U on $\mathbb{H}(\mathcal{G}/\mathcal{G}_F)$ with $\sigma(U) = g$ for some g in $Sp(\mathcal{G}/\mathcal{G}_F)$. Then σ is a surjective group homomorphism from $Mp^c(\mathcal{G}/\mathcal{G}_F)$ to $Sp(\mathcal{G}/\mathcal{G}_F)$ and we have a central short exact sequence

$$1 \to U(1) \to Mp^c(\mathcal{G}/\mathcal{G}_F) \xrightarrow{\sigma} Sp(\mathcal{G}/\mathcal{G}_F) \to 1\,, \tag{1}$$

where $U(1)$ is the group of unitary scalar operators on $\mathbb{H}(\mathcal{G}/\mathcal{G}_F)$.

If $\mathcal{G}_F = \mathcal{G}$, then taking $Mp^c(\mathcal{G}/\mathcal{G}_F) = U(1)$ we have also a central short exact sequence

$$1 \to U(1) \to Mp^c(\mathcal{G}/\mathcal{G}_F) \xrightarrow{\sigma} 1 \to 1\,, \tag{2}$$

Let $g \in G_F$ and $\widetilde{Ad}(g^{-1}) : \mathcal{G}/\mathcal{G}_F \to \mathcal{G}/\mathcal{G}_F$ be the real automorphism induced from $Ad(g^{-1}) : \mathcal{G} \to \mathcal{G}$. Then the map $j : G_F \to Sp(\mathcal{G}/\mathcal{G}_F)$ given by $j(g) = \widetilde{Ad}(g^{-1})$ is a group homomorphism from G_F to $Sp(\mathcal{G}/\mathcal{G}_F)$.

Denote by $G_F^{U(1)}$ the Lie subgroup of cartesian product $G_F \times Mp^c(\mathcal{G}/\mathcal{G}_F)$ consisting of all pairs (g, U) such that $\sigma(U) = \widetilde{Ad}(g^{-1})$, i.e. g and σ have the same image in $Sp(\mathcal{G}/\mathcal{G}_F)$. We know that every member of $G_F^{U(1)}$ has the form (g, U) such that $\sigma(U) = j(g) = \widetilde{A}g(g^{-1})$, where $U \in Mp^c(\mathcal{G}/\mathcal{G}_F)$ has parameter (λ, f), with $f \in Sp(\mathcal{G}/\mathcal{G}_F)$ and $\lambda \in \boldsymbol{C}$ such that $|\lambda^2 \det C_f| = 1$, with $C_f = \frac{1}{2}(f - if_i)$ commuting with $i \in \boldsymbol{C}$ $(i^2 = -1)$. Then

$$G_F^{U(1)} = \left\{(g;\, (\lambda,\, \widetilde{Ad}(g^{-1})))\;\middle|\; |\lambda^2 \det C_{\widetilde{Ad}(g^{-1})}| = 1\right\}$$

and we obtain the short exact sequence

$$1 \to U(1) \to G_F^{U(1)} \xrightarrow{\sigma_j} G_F \to 1\,, \qquad (3)$$

where $\sigma_j\big(g, (\lambda, \widetilde{Ad}(g^{-1}))\big) = g$. We call $G_F^{U(1)}$ a $U(1)$-covering of G_F.

It follows from (3) that we have a split short exact sequence of the corresponding Lie algebras

$$0 \to \mathcal{U}(1) \to \mathrm{Lie}G_F^{U(1)} \to \mathcal{G}_F \to 0\,. \qquad (4)$$

Thus the Lie algebra of $G_F^{U(1)}$ is $\mathcal{G}_F \oplus \mathcal{U}(1)$ (see [6, §5]).

1.2. $U(1)$-admissible orbits

Recall that the K-orbit Ω passing $F \in \mathcal{G}$ is called integral if there is a unitary character $\chi_F : G_F \to S^1$ such that $(d\chi_F)(X) = (\frac{i}{\hbar}F)(X)$, $\forall X \in \mathcal{G}_F$.

An integral orbit datum is a pair (F, π) with $F \in \mathcal{G}^*$ and π is an irreducible unitary representation of G_F on a Hilbert space V such that $d\pi = (\frac{i}{\hbar}F)\, IdV$, where IdV is the identity operator on the space V.

According to Cartan-Weyl-Kostant ([8, Theorem1]), if G is a compact Lie group then there is attached to each integral orbit datum (F, π) an irreducible unitary representation of G that is called an orbit correspondence. This orbit correspondence establishes a bijection relation between G-conjugacy classes of integral orbit data and irreducible unitary representations of G. For noncompact Lie groups, however, a nice orbit correspondence cannot be defined on integral orbit data.

Example 1.1. Let $G = \mathrm{Sp}(2n, \mathbb{R})$ be the symlectic group corresponding to the symplectic form ω in $\mathbb{R}^{2n}$ defined by

$$\omega\big((x, y), (z, t)\big) = x.t - y.z, \;\forall x, y, z, t \in \mathbb{R}^n.$$

We have

$$G=\left\{\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in M_{2n}(\mathbb{R}) \,\middle|\, A^tD-B^tC=I,\, B^tA=A^tB,\, C^tD=D^tC\right\}.$$

Its Lie algebra is

$$\mathcal{G}=\left\{\begin{pmatrix} A & B \\ C & -A^t \end{pmatrix} \,\middle|\, A,\, B,\, C\in M(n,\, \mathbb{R}),\; B^t=B,\; C^t=C\right\}.$$

The adjoint action of G given by conjugation of matrices: $Ad(g) = gXg^{-1}$.

For each $X\in\mathcal{G}$ we define a linear functional $F_X\in\mathcal{G}^*$ by $F_X(Y) = \mathrm{tr}(XY)$. Then the map $X\mapsto F_X$ is an isomorphism from $\mathcal{G}$ onto $\mathcal{G}^*$, intertwining Ad and Ad^*. Consider now the Lie algebra element

$$X=\begin{pmatrix} 0 & E_{11} \\ 0 & O \end{pmatrix}\in\mathcal{G},$$

where E_{11} is the n by n matrix with a one in the upper left corner, and all other entries zero. We write $F=F_X$.

For $n=1$, we have

$$G_F=\left\{\begin{pmatrix} \pm 1 & t \\ 0 & \pm 1 \end{pmatrix}\,\middle|\, t\in\mathbb{R}\right\},\quad \mathcal{G}_F=\left\{\begin{pmatrix} 0 & t \\ 0 & 0 \end{pmatrix}\,\middle|\, t\in\mathbb{R}\right\}.$$

Hence

$$F\begin{pmatrix} 0 & t \\ 0 & 0 \end{pmatrix}=\mathrm{tr}\left(\begin{pmatrix} 0 & t \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & t \\ 0 & 0 \end{pmatrix}\right)=0.$$

Then the restriction of F to $\mathcal{G}_F$ is 0.

The general case $n\geq 2$ is similar. It follows that the K-orbit Ω passing F in integral, and $(F,\, \pi)$ is an integral orbit datum if we take π to be the trivial representations of G_F. Nevertheless, it can be show that for $n\geq 2$, there is no irreducible unitary representation of $\mathrm{Sp}(2n,\, \mathbb{R})$ attached to $(F,\, \pi)$ in any reasonable sense (see [8]). □

We therefore need something slightly different from integral orbit data to get a nice orbit correspondence.

Using metaplectic coverings (two-fold coverings) of stabilizer, M. Duflo and D. Vogan proposed $\boldsymbol{Z}_2$-admissible orbit data to get a nice orbit correspondence for reductive Lie groups.

We will suggest here $U(1)$-admissible orbit data by using Mp^c-structures instead of metaplectic structures and point out a nice bijection corespondence between $U(1)$-admissible orbits and $\mathbf{Z}_2$-admissible orbits.

Definition 1.1. The K-orbit Ω passing F is called $U(1)$-integral if there exists a unitary character

$$\chi_{F,k}^{U(1)} : G_F^{U(1)} \to S^1 ,$$

such that

$$(d\chi_{F,k}^{U(1)})(X, \varphi) = \frac{i}{\hbar}(F(X) + k\varphi) ,$$

where $(X, \varphi) \in \mathcal{G}_F \oplus \mathcal{U}(1)$ and $k \in \mathbf{Z}$.

Remark. If Ω is an integral orbit then it is $U(1)$-integral, but the converse does not holds. For $k \neq 0$, it is enough to consider the case $k = 1$. The orbit then is called $U(1)$-admissible and we denote simply $\chi_{F,1}^{U(1)} = \chi_F^{U(1)}$.

Proposition 1.1. ([7, §2]) *In the neighbourhood of the identity of $G_F^{U(1)}$ we have*

$$\chi_F^{U(1)}(g, (\lambda, \widetilde{Ad}(g^{-1})) = exp\{\frac{i}{\hbar}(F(X) + \varphi)\} ,$$

where $\varphi \in \mathbb{R}$ satisfies the relation $\lambda^2 detC_{\widetilde{Ad}(g^{-1})} = e^{\frac{i}{\hbar}\varphi}$. The integral kernel of $\chi_F^{U(1)}$ is given by the formula

$$u(z, \omega) = exp\{\frac{i}{\hbar}(F(X) + \varphi) + \frac{1}{2\hbar}\langle z, w\rangle - \frac{1}{4\hbar}\langle w, w\rangle\},$$

where $z, w \in (\mathcal{N} + \overline{\mathcal{N}})/(\mathcal{N} \cap \overline{\mathcal{N}})$, with $\mathcal{N}$ is a positive polarization in $\mathcal{G}_{\mathbf{C}}$.

2. THE RELATION BETWEEN $U(1)$-COVERINGS AND TWO-FOLD COVERINGS

Consider the unitary character (see [5, §2])

$$\eta : Mp^c(\mathcal{G}/\mathcal{G}_F) \to U(1)$$

$$U \mapsto \eta(U) = \lambda^2 \det C_f ,$$

where $U \in Mp^c(\mathcal{G}/\mathcal{G}_F)$ has parameter (λ, f).

The kernel $Mp(\mathcal{G}/\mathcal{G}_F)$ of η is called the metaplectic group. Since η restricts to $U(1) \subset Mp^c(\mathcal{G}/\mathcal{G}_F)$ as a squaring map, we obtain a central short exact sequence

$$1 \to \boldsymbol{Z}_2 \to Mp(\mathcal{G}/\mathcal{G}_F) \to Sp(\mathcal{G}/\mathcal{G}_F) \to 1$$

and $Mp(\mathcal{G}/\mathcal{G}_F)$ is a connected two-fold covering of $Sp(\mathcal{G}/\mathcal{G}_F)$.

Proposition 2.1. *We have the following short exact sequence*

$$1 \to U(1) \xrightarrow{\kappa} (Mp(\mathcal{G}/\mathcal{G}_F) \times U(1))/\langle(-\mathbf{1}, 1)\rangle \xrightarrow{\pi} Sp(\mathcal{G}/\mathcal{G}_F) \to 1,$$

where $-\mathbf{1} = (-1, I) \in \boldsymbol{C}^* \times Sp(\mathcal{G}/\mathcal{G}_F)$ *and* $\langle(-\mathbf{1}, 1)\rangle \cong \boldsymbol{Z}_2$.
In other words, we have

$$(Mp(\mathcal{G}/\mathcal{G}_F) \times U(1))/\langle(-\mathbf{1}, 1)\rangle \cong Mp^c(\mathcal{G}/\mathcal{G}_F) .$$

Proof. Define

$$\kappa(u) = (\mathbf{1}, u)\langle(-\mathbf{1}, 1)\rangle, \ \forall u \in U(1)$$

$$\pi\big(((z, p), u)\langle(-\mathbf{1}, 1)\rangle\big) = p, \ \forall((z, p), u) \in Mp(\mathcal{G}/\mathcal{G}_F) \times U(1) .$$

We see easily that κ is injective and is surjective. Moreover,

$$\big(((z, p), u)\langle(-\mathbf{1}, 1)\rangle\big) \in \operatorname{Ker}(\pi) \Leftrightarrow \det C_p = z^2 \text{ and } p = I.$$

In other words, $z^2 = 1$ and $p = I$;
hence $((z, p), u)\langle(-\mathbf{1}, 1)\rangle = (\mathbf{1}, u)\langle(-\mathbf{1}, 1)\rangle$.
It follows $\operatorname{Ker}(\pi) = \operatorname{Im}(i)$, and then the above sequence is exact. □

According to M. Duflo [3], the K-orbit passing $F \in \mathcal{G}^*$ is called $\boldsymbol{Z}_2$-admissible if the (unitary) character

$$\chi_F(\exp X) = \exp\{\frac{i}{\hbar}F(X)\}$$

of the indentity component $(G_F)_0$ can be lifted to a character of the two-fold covering $G_F^{\boldsymbol{Z}_2}$ of G_F such that $\chi_F(1, \epsilon) = -1$, where ϵ is the generator of $\boldsymbol{Z}_2$.

A representation of (π, V) of $G_F^{\mathbb{Z}_2}$ is called genuine if $\pi(1, \epsilon) = -Id_V$.

An $\mathbb{Z}_2$-admissible orbit datum is a pair (F, π) with $F \in \mathcal{G}^*$ and π is a genuine irreducible unitary representation of $G_F^{\mathbb{Z}_2}$ on a Hilbert space V such that $d\pi = \left(\frac{i}{\hbar}\right) Id_V$.

Proposition 2.2. *There exists a bijection relation between $U(1)$-admissible orbits and $\mathbb{Z}_2$-admissible orbits.*

Proof. Suppose that the K-orbit Ω passing F is $U(1)$-admissible, we show that Ω is $\mathbb{Z}_2$-admissible.

Indeed, we firstly find non trivial elements in kernel of the projection σ_j corresponding to

$$1 \to U(1) \to G_F^{U(1)} \overset{\sigma_j}{\to} G_F \to 1\,,$$

$$G_F^{U(1)} = \left\{ \left(g;\, (\lambda,\, \widetilde{A}d(g^{-1}))\right) \Big|\, |\lambda^2 \det C_{\widetilde{A}d(g^{-1})}| = 1 \right\}.$$

The unit element of $G_F^{U(1)}$ is $e = (1;\, (1,\, I))$. Choose a non trivial element in $\operatorname{Ker}\sigma_j$ is $(1;\, (-1,\, I))$ to have

$$\chi_F^{U(1)}(1;\, (-1,\, I)) = -1\,.$$

Then Ω is $\mathbb{Z}_2$-admissible.

Note that the non trivial elements in $\operatorname{Ker}\sigma_j$ are

$$(1;\, (\lambda,\, I)) \text{ with } |\lambda|^2 = 1,\ \lambda \neq 1$$

and we have by definition

$$\chi_F^{U(1)}(1;\, (\lambda,\, I)) = \lambda\,.$$

Thus it is satisfied the definition of M. Duflo.

We suppose now that Ω is $\mathbb{Z}_2$-admissible. There exists by definition a unitary character χ_F of $G_F^{\mathbb{Z}_2}$ such that for $X \in \mathcal{G}_F$ we have

$$\chi_F(1,\, \epsilon) = -1 \text{ and } \left(d\chi_F\right)(X) = \frac{i}{\hbar} F(X)\,.$$

Using the isomorphism in Proposition 2.1 we can extend χ_F to a character $\chi_F^{U(1)}$ of $G_F^{U(1)} \cong G_F^{\mathbf{Z}_2} \times U(1)/\langle(-\mathbf{1}, 1)\rangle$ such that

$$(d\chi_F^{U(1)})(X, \varphi) = \frac{i}{\hbar}(F(X) + \varphi),$$

where $(X, \varphi) \in \mathcal{G}_F \oplus \mathcal{U}(1)$. In other words, the orbit Ω is $U(1)$-admissible. □

Example 2.1. Let $G = SL(2, \mathbb{R})$. We identify $\mathcal{G}$ and $\mathcal{G}^*$ with two by two matrices of trace 0 by using the trace form: if F is a matrice in $\mathcal{G}^*$ and X is a matrice in $\mathcal{G}$, then $F(X) = \mathrm{tr}(FX)$. The coadjoint action is then given by conjugation of matrices and we have

$$\mathrm{tr}(FAXA^{-1}) = \mathrm{tr}(A^{-1}FAX)\,, \ \forall A \in G,\ X \in \mathcal{G}^*,$$

so G_F consists of all matrices in G commuting with F.

Set $F = \begin{pmatrix} t & 0 \\ 0 & -t \end{pmatrix}$, with $t > 0$, we have

$$G_F = \left\{ \begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix} \middle| \, a \in \mathbb{R}^* \right\}.$$

We therefore identify G_F with $\mathbb{R}^*$. Consequently

$$G_F^{\mathbf{Z}_2} = \left\{(a, z) \in \mathbb{R}^* \times \mathbf{C} \,\middle|\, a^2 = z^2\right\} \cong \mathbb{R}^* \times \{\pm 1\}$$

and

$$G_F^{U(1)} = \left\{(a, z) \in \mathbb{R}^* \times \mathbf{C} \,\middle|\, a^2 = |z|^2\right\} \cong \mathbb{R}^* \times U(1)\,.$$

There are two $U(1)$-admissible characters of $G_F^{U(1)}$ induced from the $\mathbf{Z}_2$-admissible characters of $G_F^{\mathbf{Z}_2}$ given by

$$\chi_{F,1}(a, \epsilon) = \epsilon|a|^{it} \text{ and } \chi_{F,2}(a, \epsilon) = \epsilon|a|^{it}\mathrm{sign}(a)\,.$$

Inparticular, the orbit Ω passing F is $U(1)$-admissible for all $t > 0$.

Example 2.2. Suppose again $G = \mathrm{Sp}(2n, \mathbb{R})$. As in Example 1.1, we can identify $\mathcal{G}$ with $\mathcal{G}^*$. The coadjoint action is then by conjugation of matrices.

Let $F = F_X$, the Lie algebra element corresponding to

$$X = \begin{pmatrix} 0 & E_{11} \\ 0 & O \end{pmatrix} \in \mathcal{G},$$

considered in Example 1.1.

If $n = 1$, we have

$$G_F \cong (G_F)^0 \times \{\pm Id\},$$

where $(G_F)^0 \cong \mathbb{R}$. Then

$$G_F^{\mathbb{Z}_2} \cong \mathbb{R} \times \{1, \epsilon\} \times \{\pm Id\} \text{ and } G_F^{U(1)} \cong \mathbb{R} \times U(1) \times \{\pm Id\}.$$

Consequently F is $U(1)$-admissible.

If $n > 1$, then $(G_F)^0$ contains $\mathrm{Sp}(2n-2, \mathbb{R})$ and the preimage of $(G_F)^0$ in $Mp(2n, \mathbb{R})$ is naturally isomorphic to the connected group $Mp(2n-2, \mathbb{R})$. Hence ϵ belongs to the identity component of $G_F^{\mathbb{Z}_2}$, and it follows that F is not $\mathbb{Z}_2$-admissible, that is F is not $U(1)$-admissible (see [8]). This is consistent with the claim in Example 1.1 that there is no irreducible unitary representation attached to F.

An $U(1)$-admissible orbit datum is a pair (F, π) with $F \in \mathcal{G}^*$ and π is a irreducible unitary representation of $G_F^{U(1)}$ on a Hilbert space V such that

$$(d\pi)(X, \varphi) = \frac{i}{\hbar}(F(X) + \varphi)\, Id_V, \quad (X, \varphi) \in \mathcal{G}_F \oplus \mathcal{U}(1).$$

It follows from [8, Theorem 3] we have

Theorem 2.1. *Suppose G is a real redutive Lie group and (F, π) is a $U(1)$-admissible orbit datum. If F is a regular semisimple element (i.e., if $\mathcal{G}_F$ is a Cartan subalgebra), then there is attached to this datum an irreducible unitary tempered representation having regular infinitesimal character. This established a bijection between the G-conjugacy classes of regular semisimple $U(1)$-admissible orbit data and the irreducible unitary tempered representations of regular infinitesimal character.*

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Department of Mathematics
College of Teacher education, Hue University
Hue, Vietnam.