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A Short Communication

A REMARK ON RIEMANN HOLOMORPHIC EXTENSION PROPERTY[1]

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It is well–known [1] that every holomorphic function on a normal complex space can be holomorphically extended through analytic sets of codimension ≥ 2. In general case this result is not true. In this note we shall investigate the Riemann holomorphic extendability of complex Lie groups.

We say that a complex space X has the Riemann holomorphic extension property in the dimension n if every holomorphic map from $Z \setminus S$, where Z is a normal complex space of dimension n and S is an analytic set in Z of codimension ≥ 2, to X can be holomorphically extended on Z.

We shall prove the following:

Theorem. *Let G be a complex Lie group of dimension 2. Then G has the Riemann holomorphic extension property in the dimension 2 if and only if G does not contain a compact curve.*

Proof. (i) Assume that G does not contain a proper compact analytic set of positive dimension. Given $f : Z \setminus S \to G$ a holomorphic map, where Z is a normal complex space with $\dim Z = \dim G$ and S is an analytic set of codimension ≥ 2. Take a plurisubharmonic exhaustion φ on G. Such a function exists by [4]. Then φf is plurisubharmonic on Z [2]. By [6] we can find a holomorphic bundle map $\theta : G \to T$ with the fibers that are Stein manifolds, where T is a complex torus. Since T is a compact Kahler manifold, by [5] θf can be extended to a meromorphic map $g : Z \to T$. Hence if $\gamma : \widehat{Z} \to Z$ is the Hironaka singular resolution of Z, then $h = g\gamma$ is holomorphic on $\widehat{Z}$. For each $z^0 \in \gamma^{-1}(S)$ take two neighbourhoods U and V of z^0 and $h(z^0)$ respectively such that $h(U) \subset V$ and $\theta^{-1}(V)$ is a Stein manifold. Then $f\gamma(U \setminus \gamma^{-1}(S)) \subset \theta^{-1}(V)$. By the upper semicontinuity of $\varphi f \gamma$ on $\widetilde{Z}$ and since φ is an

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exhaustion function on G, it follows that $f\gamma|_{U\setminus\gamma^{-1}(S)}$ can be extended holomorphically on $\widehat{Z}$. Hence f can be extended to a meromorphic map $\hat{f}: Z \to G$. Put $\widetilde{I}(\hat{f}) = \{z \in Z : \hat{f}(z) = G\} \subseteq I(\hat{f})$.

By the hypothesis, $\dim Z = \dim G$. Therefore, by the density of $\Gamma(\hat{f}) \setminus \sigma_{\hat{f}}^{-1}(I(\hat{f}))$ in $\Gamma(\hat{f})$ and the properness of the projection $\sigma_{\hat{f}}$: $\Gamma(\hat{f}) \to Z$ it follows that $\widetilde{I}(\hat{f}) = \emptyset$. Thus $\hat{f}(z)$ is a proper analytic compact set in G for every $z \in I(\hat{f})$. By the hypothesis, $\dim \hat{f}(z) = 0$ for every $z \in I(\hat{f})$. This implies, from the normality of Z, the holomorphicity of $\hat{f}$.

Now assume that G has the Riemann holomorphic extension property in the dimension 2. Let G contains a proper compact analytic set X of positive dimension. Since $\dim X = 1$, it follows that X is projective. Thus there exists a normal cone $\widetilde{X}$ over X. Obviously the canonical map $\widetilde{X} \setminus \{0\} \to X$ cannot be extended holomorphically at 0.

Problem. Let G be a complex Lie group of any dimension. Prove that G has the Riemann holomorphic extension property in its dimension if and only if G does not contain a compact curve.

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