VIETNAM JOURNAL OF MATHEMATICS
Volume 23, Number 2, 1995

MATHEMATICS AND DEVELOPMENT[1]

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I am deeply moved by the honour that has fallen to my lot to received and I am happy to have the opportunity to talk on some specific problems concerning mathematics and development, before such an honorable audience.

What is the role of mathematics in the industrialization process is a theme of considerable interest both for people from the South and the North, inside and outside the mathematical community. In fact, in our time, development – the biggest problem of major part of humanity – has to be achieved in the context of extremely rapid technological changes, most of them directly or indirectly connected with mathematics which has become increasingly vital to science, technology and society.

Not without reason has the International Union of Mathematicians announced recently the proposal on the World Mathematical Year 2000. One of the main arguments of this proposal is that, in a world quickly moving towards the age of information highway, mathematics should be considered as a key to development. Every people concerned with the progress of science and the problems the society is facing in the transition to the new era should be aware of the impact that the mathematization process will have on our life and work in the next century.

However, it is beyond my competence and my aim to discuss every aspect of this important theme. Rather, I will confine myself to topics which are nearer to me, concentrating on my own viewpoints and illustrating them by my personal experience wherever possible. So I will mainly touch on the parts of mathematics I have been dealing with in the past three decades, namely: operations research, mathematical

[1]From the Editors: Professor Hoang Tuy, former chief-editor of our journal (1980 – 1990) and former director of the Hanoi Institute of Mathematics (1980 – 1990), was awarded the Honorary Doctor Degree at Linköping University (Sweden), 2nd June 1995. The present article is the award lecture he delivered on this occasion.

economics and optimization. Even within these limits I will be able to discuss only a few relevant points.

By force of circumstance, I have been from youth nearly always living, studying, and working in a poor backward country which, on top of this, in a long period of time (about 35 years) had very few relations with the outside, except for the frequent cluster bombs received (if one counts these as a form of relations, as the French mathematician A. Grothendieck wrote in his article about his visit to our University in 1967). It is natural under these conditions that, when the country returns to the normal life, the fight against poverty and the struggle for a sustainable development must be the central task, the number one problem of every citizen. On the other hand, I am a mathematician with, basically, the same professional responsibilities and the same concerns as mathematicians everywhere else. That sounds a bit incompatible, at least unusual, because it is not so evident that one can reconcile the principle of academic freedom with the need to study problems perceived by target groups and policy makers. Quite often the question arises as to whether mathematical sciences (and other fundamental sciences) are not luxuries for developing countries? Can mathematics and mathematicians really contribute to the economic development? Is it reasonable, is it worthwhile and at all possible to do serious mathematical researches in an economically backward environment?

Indeed, answers to these questions should be of special interest for many countries in the developing world.

In highly industrialized countries it is generally perceived that the prosperity of the society very much depends on the development of mathematics as the foundation of science and engineering, and that excellence in mathematics is a condition for excellence in technology. Over the past few decades, many new mathematical applications have literally exploded, due to the availability of very fast computers with large memories. In particular, the use of mathematical modeling to replace experimentation for design and the ability to generate and store unprecedented huge amounts of data from which hidden information can be extracted through mathematical manipulations have let to spectacular innovations in science and technology. Mathematics is expected to have even more significant impact over the entire spectrum of the next round or the technological revolution, from establishing new communication systems to software development for their exploitation and management.

However, one may object that all this may be true only for highly industrialized societies, with large-scale industries at an advanced level of technology, whereas the problems facing developing countries are dramatically different: poor infrastructure, risky food production, poor market structure, lack of credit systems, etc... How mathematics can be useful for the solution of such problems?

To answer this issue properly, one should bear in mind that developing countries should not repeat the entire historical industrialization process of the developed countries. Very often, short cuts are not only possible but even imperative, as demonstrated by the examples of Japan a hundred years ago and the new industrialized countries in the last few decades. There is no reason that problems in a backward economy could only be solved by backward methods. Therefore, advanced methods, sometimes with sophisticated mathematical foundation, can be and have in fact been used in India, China, South Corea, etc... to solve problems which long ago were already solved in developed countries, but by methods which now have become absolete. Let me quote the application of operations research, statistical methods (in particular, statistical quality control), mathematical modeling, optimization, numerical analysis, etc... which have made sometimes very valuable contribution to the study and solution of problems in agriculture, ecology, environment, water resources management, flood control, filtration, energy problems, oil industry. Thirty years ago, no one would have predicted that hightech has now penetrated so solidly in certain aspects of everyday life in some southeast asian countries. Talking about infrastructures, economists estimate that in the next twenty years, huge amounts of money of the order of several hundreds billions of dollars will be needed in South-East Asia just for the building up and modernization of necessary infrastructure for a further accelerated economic development of this part of the world which not so long ago was sometimes considered as condemned to incurable poverty.

As early as in the late fifties, the famous chinese mathematician Hua Loo-Keng initiated a vast campaign for the application of operations research and optimization in China. These activities have been well reported in a recent article by Schweigman and Zhang in Mathematical Intelligencer, 1994. In Vietnam we had similar experiences, although at a much smaller scale: linear programming and statistical methods were applied in the transportation sector (for instance to the problem of reorganizing the logistics of trucking so as to reduce the distance that trucks travel empty), in agriculture, (to maximize expected crops, or increase farmer's income), in textile industry, in construction and project management (citical path method), etc... Though the results were sporadic and many of the new methods which had been successfully implemented were later abandoned because of bad management and the lack of incentives, these experiences demonstrated the relevance and the practicability of the mentioned mathematical methods even in low-tech situations.

In a world in which productivity heavily depends on quantitative literacy of the work force, the development of mathematics is not only imperative for countries which want to enhance competitiveness of their economy but also necessary for poorer countries as well, if these want to close the gap between them and the rich countries in the shortest time. To produce a well trained work force, the education system must be adequate. Since mathematics at college and universities form the core of the quantitative skills necessary for the scientific, technical and managerial work force, highly qualified mathematics teachers at every level are needed. However, teaching cannot be separated from research, especially at the university level. Therefore, mathematical research is not a luxury even in developing countries but should be promoted starting now, without waiting for a better future. The main issue is only how to carry out research in order to best use the limited resources available.

Of course the problem is far from simple. One should be aware of the enormous and innumerable difficulties of mathematical research in the absence of adequate supporting infrastructure, academic climate and environment. However, with all the changes in recent years, mathematics continues to be a "small" science, requiring a minimum equipment, as compared to other sciences: with paper, pencil, a computer and a good brain one can sometimes do a lot of valuable mathematics. Excellence in mathematics is a necessary condition, though of course not sufficient, for economic development. The experiences of Hungary, Poland, India, Brazil, are precious in that they show the capability of a country to produce outstanding mathematical schools, even without a corresponding maturity in economic development.

In the case of Vietnam, I should say that our efforts to link mathematics with development, although not always successful, have been beneficial to the development of mathematics itself. The fact that under extremely hard conditions, even in the most difficult period of crisis, we were able to hold regular seminars on various fields of mathematics, the consistently good performance of our team at international mathematical olympiads, the healthy growth of our research institute of mathematics, all these facts are very encouraging, in spite of some serious recent problems resulting from the mismanagement of the education system.

For me and my colleagues, all these activities in the application fields were very instructive and inspiring.

I started my scientific career some fourty years ago as a researcher in real analysis and functional analysis. Therefore, when I began studying operations research, at first I was not very pleased with the type of mathematics that was used. The first text of linear programming available to me was not a good one and I must say that the field struck me as boring in comparaison with the beauty and elegance of, say, measure theory or functional analysis, which were more aesthetically satisfying to me. But my commitments with practical applications eventually led me to leave real analysis and shift definitively to optimization, especially after the advices I received from Kantorovich when I visited him in 1962.

At that time we were engaged in the application of linear programming methods to transportation problems. I first became interested in problems with convex nonlinear costs, but soon I realized that convexity was an assumption convenient for the mathematicians but unrealistic in many situations due to economies of scale and fixed charges. This was the origin of my work on minimum concave cost transportation problem, or more generally, the problem of minimizing a concave function under linear constraints, published in Soviet Doklady in 1964, which appeared to be one of the first steps in deterministic global optimization. It was in this paper that the concavity cut and conical partition, subsequently to become of frequent use in global optimization, were introduced.

Later, in connection with the study of the functioning mechanism of the economy, I became interested in the mathematical nature of such concepts as optimal prices, dual variables, Lagrange multipliers, decomposition, decentralization, equilibrium, and the like. This motivated my work on convex inequalities, minimax theorems and fixed point theorems. In all these basic theorems, I saw the fundamental role of convexity. However, more and more nonconvex problems appear which cannot be handled by traditional methods of local optimization because of the existence of many local minima which fail to be global. As I mentioned earlier, concavity of costs due to the economy of scale and fixed charges arises, for instance, in production–transportation and plant location problems. Since my primary formation was set-theoretical and functional analytic, I wanted to look into the general analytical structure underlying all these nonconvex problems. Fortunately, close scrutiny reveals that in the analysis of nonconvex problems convexity plays a no less important role that in the analysis of traditional convex problems, only most of the time in a quite different way. The profound reason is that the topology of the space R^n in which we are moving is such that any arbitrary closed set in it is the projection of a difference of two convex sets in a space of one dimension higher.

This allows us to describe any continuous optimization problem over a compact set as a problem of minimizing or maximizing a linear function over a d.c. set, i. e. a set which is equal to the difference of two convex sets. This was the beginning of a new chapter of deterministic global optimization, called d.c. optimization.

The development of this new field of research in the last decade has highlighted several important achievements, both from a theoretical and a computational practical point of view. Along with far-reaching theoretical results concerning duality, decomposition, low rank nonconvex structures, I would like to mention the new fields of applications: engineering design, network optimization, production – transportation problems involving both economies and diseconomies of scale (increasing and decreasing returns), bilevel programming, multiobjective programming, financial management problems, continuous location and most recently, computational biology and computational chemistry.

To conclude, despite many challenging difficulties ahead, I think, we can be optimistic about the prospects. Mathematics, when linked to development, even if unsuccessful for solving today's problems, can at least enrich our knowledge reserve, hopefully, for future use, and therefore, can save us from the bitter feeling of dissatisfaction as that conveyed by Hardy's celebrated book "A Mathematician's Apology".

Received September 28, 1995

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