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A Short Communication

RINGS WHOSE FINITELY GENERATED MODULES ARE EXTENDING

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Let R be a ring and M be a right R-module. Then M is called an extending (or CS) module if every submodule of M is essential in a direct summand of M. For a detailed study of extending modules we refer to [1].

The main result of this paper is the following

Theorem 1. *Let M be a right module over a given ring R. If every finitely generated M-singular module is extending, then $M/Soc\ (M)$ is locally noetherian, where $Soc\ (M)$ is the socle of M.*

From Theorem 1 it follows

Corollary 2. *A right R-module M is locally noetherian if every finitely generated module in $\sigma[M]$ is a direct sum of an M-projective module and an extending module.*

For $M = R$ we get an answer for an open question raised in [2], for a related study of which we refer to [3]: A ring R is right noetherian if every 2-generated right R - module is extending.

Applying Theorem 1 we obtain the following characterization of some serial artinian rings as follows:

Theorem 3. *For a ring R the following conditions are equivalent.*

(a) *The injective hull of R_R is finitely generated and any 2-generated right R-module is extending.*

(b) *Every right R-module is extending.*

(c) *Every countably generated right R-module is extending.*

(d) *R is right and left artinian, right and left serial with $J(R)^2 = 0$.*

(e) *The left-handed versions of (a), (b) and (c).*

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