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p-GROUPS WITH CYCLIC FRATTINI SUBGROUP

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Abstract. *Let G be a p-group and let $Z(G)$, $\Phi(G)$ be the center and Frattini subgroup of G. We give cohomological proofs of Hobby's theorem, which asserts that $\Phi(G)$ is cyclic if $Z(\Phi(G))$ is cyclic, and of Berger, Kovac's and Neumann's result on the classification of p-groups with cyclic Frattini subgroup.*

INTRODUCTION

Let p be a prime number. For every p-group G, let $Z(G)$ and $\Phi(G)$ be respectively the center of Frattini subgroup of G. The purpose of this work is to give cohomological proofs of Hobby's theorem [2] (III 7.8 c in [3]), which asserts that $\Phi(G)$ is cyclic if $Z(\Phi(G))$ is cyclic, and of Berger, Kovac's and Neumann's result [1] on the classification of p-groups with cyclic Frattini subgroup.

Following [1], directly and centrally indecomposable groups with cyclic Frattini subgroup can be obtained as follows. First, one gets two factor groups of

$$L = \langle a, b, c \mid a^{2^{\ell+1}} = b^2 = 1,\ a^b = a^{-1+2^\ell},\ a^c = a^{1+2^\ell},\ bc = cb\rangle$$

with $\ell \geq 1$ by setting $D^+(2^{\ell+3}) = L/\langle c^2\rangle$, $Q^+(2^{\ell+3}) = L/\langle c^2.a^{2^\ell}\rangle$. Secondly, one has the cyclic group $C_{p^{\ell+2}}$ of order $p^{\ell+2}$, the extra-special p-groups of order p^3, and the non-abelian groups with cyclic maximal subgroups of order greater than p^3. The latter consist of

(1) $M(p^{\ell+2}) = \langle a, b \mid a^{p^{\ell+1}} = b^p = 1,\ a^b = a^{1+p^\ell}\rangle$ for $p > 2$,

(2) $M(2^{\ell+2})$, $D(2^{\ell+2}) = \langle a, bc\rangle \subset D^+(2^{\ell+3})$, $S(2^{\ell+2}) = \langle a, b\rangle \subset D^+(2^{\ell+3})$,
$Q(2^{\ell+2}) = \langle a, bc\rangle \subset Q^+(2^{\ell+3})$ for $p = 2$
(especially, $D(8) = M(8)$, $S(8) = C_4 \times C_2$).

As we shall see in Section 1, all these groups can be defined as extensions of Z_2 by either an abelian group, or $D(2^{\ell+1})$, or $D(2^{\ell+1}) \times C_2$.

In Section 2 we give a cohomological proof of the following

Theorem A (Hobby [2]). *If $Z(\Phi(G))$ is cyclic, then so is $\Phi(G)$.*

Theorem B (Berger, Kovac's, Neumann [1]). *Let G be a p-group with cyclic Frattini subgroup of order p^ℓ. If $|G| = p^{n+\ell}$, then G is isomorphic to one of the following groups*

$$C_{p^{\ell+1}} \cdot \underbrace{E \cdot \ldots \cdot E}_{m-1 \ times} \times C_p^{n-2m+1}, \quad M \cdot \underbrace{E \cdot \ldots \cdot E}_{m-1 \ times} \times C_p^{n-2m},$$

$$N \cdot \underbrace{E \cdot \ldots \cdot E}_{m-2 \ times} \times C_2^{n-2m+1}, \quad D^+(2^{\ell+3}) \cdot C_4 \cdot \underbrace{E \cdot \ldots \cdot E}_{m-2 \ times} \times C_2^{n-2m}.$$

Here and in what follows, $A \cdot B$ means the central product of A and B with $|A \cap B| = p$, M is one of the groups given by (1) and (2) if $\ell > 1$, or an extra-special p-group if $\ell = 1$, N is either $D^+(2^{\ell+3})$, $Q^+(2^{\ell+3})$, $D(2^{\ell+2}) \cdot C_4$ or $S(2^{\ell+2}) \cdot C_4$ and

$$E = \begin{cases} \langle a, b | a^p = b^p = [a,b]^p = [a,[a,b]] = [b,[a,b]] = 1 \rangle & \text{if } p > 2 \\ D(8) & \text{if } p = 2. \end{cases}$$

1. PRELIMINARIES

For every p-group K, let us denote by $H^*(K)$ the mod p cohomology algebra of K. Let a be a generator of the cyclic group C_{p^ℓ} and let u_a, v_a be respectively the 1- and 2-cocycles of C_{p^ℓ} given by

$$u_a(a) = 1, \quad v_a(a^i, a^j) = \begin{cases} 0, & \text{if } i + j < p^\ell \\ 1, & \text{otherwise.} \end{cases}$$

So $v_a = \beta u_a$, for $\ell = 1$, with β the Bockstein homomorphism. It is well-known that

$$H^*(C_{p^\ell}) = \begin{cases} P[u_a; 1], & \text{if } \ell = 1 \text{ and } p = 2 \\ E[u_a; 1] \otimes P[v_a; 2], & \text{otherwise.} \end{cases} \tag{1.1}$$

Here and in what follows, $E[x, y, \ldots; 1]$ (resp. $P[x, y, \ldots; 2]$) means the exterior (resp. polynomial) algebra over Z_p of generators $x, y, \ldots$ of degree 1 (resp. 2).

Hence, if k is an integer and $C_{p^\ell} \times C_p^{k-1} = \langle a_1, \dots, a_k | a_1^{p^\ell} = a_i^p = [a_j, a_i] = 1, 2 \le i \le k, 1 \le j \le k \rangle$, by setting $u_i = u_{a_i}$, $v_i = v_{a_i}$, we get

$$H^*(C_{p^\ell} \times C_p^{k-1}) = \begin{cases} P[u_1, \dots, u_k; 1], & \text{if } \ell = 1 \text{ and } p = 2 \\ E[u_1; 1] \otimes P[v_1; 2] \otimes P[u_2, \dots, u_k; 1], & \text{if } \ell > 1 \text{ and } p = 2 \\ E[u_1, \dots, u_k; 1] \otimes P[v_1, \dots, v_k; 2], & \text{if } p > 2. \end{cases}$$

The following lemma is then obvious.

Lemma 1.2. *Let* $0 \ne X \in H^2(C_{p^\ell} \times C_p^{k-1})$. *Assume furthermore that* $\mathrm{Res}(\langle a_1^{p^{\ell-1}} \rangle, C_{p^\ell} \times C_p^{k-1})X = v_{a_1^{p^{\ell-1}}}$ *if* $\ell > 1$. *Then* X *can be reduced by an automorphism of* $C_{p^\ell} \times C_p^{k-1}$ *to one of the following forms*

i) $\sum_{i=1}^{m} u_{2i-1} \cdot u_{2i}$, $u_1^2 + \sum_{i=2}^{m} u_{2i-1} \cdot u_{2i}$, $u_1^2 + u_2^2 + u_1 \cdot u_2 + \sum_{i=2}^{m} u_{2i-1} \cdot u_{2i}$ *if* $p = 2$ *and* $\ell = 1$;

ii) $\lambda v_1 + \mu u_1 \cdot u_2 + \sum_{i=2}^{m} u_{2i-1} \cdot u_{2i}$ *if* $\ell > 1$ *or* $p > 2$, *with* $\mu = 0$ *or 1, and*

$$\lambda = \begin{cases} 1, & \text{if } \ell > 1 \\ 0 \ \text{ or } \ 1, & \text{otherwise.} \end{cases}$$

Hence, if $\mathrm{Res}(\langle a \rangle, C_{p^\ell} \times C_p^{k-1})X \ne 0$, *for every* $a \in C_{p^\ell} \times C_p^{k-1}$, *then* $k = 1$.

Note that the factor sets of the central extensions

$$0 \to Z_p \to C_{p^{\ell+1}} \to C_{p^\ell} \to 1,$$
$$0 \to Z_p \to G \to C_{p^\ell} \times C_p \to 1$$

with $G = E$, $Q(8)$ (for $\ell = 1$), $M(p^{\ell+2})$ are respectively

$$v_1, \quad u_1 \cdot u_2, \quad u_1^2 + u_2^2 + u_1 \cdot u_2, \quad v_1 + u_1 \cdot u_2$$

(see e.g. [4] or [5], [6], [8]). We have

Lemma 1.3. *Let* $0 \to Z_p \to G \to C_{p^\ell} \times C_p^{k-1} \to 1$ *be a central extension with factor set* $0 \ne z \in H^2(C_{p^\ell} \times C_p^{k-1})$ *having one of the forms given in Lemma 1.2. Then* G *is isomorphic to one of the following groups*

$$C_{p^{\ell+1}} \cdot \underbrace{E \cdot \ldots \cdot E}_{m-1 \text{ times}} \times C_p^{k-2m+1}, \quad M \cdot \underbrace{E \cdot \ldots \cdot E}_{m-1 \text{ times}} \times C_p^{k-2m}.$$

Analogous results can be stated if we replace C_{p^ℓ} by $D(2^\ell)$. Recall that

$$D(2^\ell) = \langle a, b \mid a^{2^{\ell-1}} = b^2 = 1,\ a^b = a^{-1}\rangle.$$

Let u_a, u_b be elements of $H^1(D(2^\ell))$ given by $u_a(a^i b^j) = i$, $u_b(a^i b^j) = j$ for $0 \le i < 2^{\ell-1}$, $0 \le j < 2$, and $z_\ell \in H^2(D(2^\ell))$ be the factor set of the central extension $0 \to Z_2 \to D(2^{\ell+1}) \to D(2^\ell) \to 1$. The following is due to Quillen [7] and Mui [6].

Lemma 1.4. *$H^*(D(2^\ell)) = P[u_a, u_b, z_\ell]/(u_a^2 + u_a \cdot u_b)$. Furthermore, we have:*

(i) $\beta z_\ell = u_b \cdot z_\ell$.

(ii) if $A = \langle a^{2^{\ell-2}},\ a^i b\rangle$ is a maximal elementary abelian subgroup of $D(2^\ell)$, with $0 \le i < 2^{\ell-1}$, then $\mathrm{Res}(A, D(2^\ell)) z_\ell = u^2_{a^{2^{\ell-2}}} + u_{a^{2^{\ell-2}}} \cdot u_{a^i b}$.

Let c be a generator of C_2. Set $\Gamma = D(2^\ell) \times C_2$, we have

Lemma 1.5. *Let $X \in H^2(\Gamma)$ with $\mathrm{Res}(\langle a^{2^{\ell-2}}\rangle, \Gamma) X = u^2_{a^{2^{\ell-2}}}$. Then X can be reduced by an automorphism of Γ to one of the following forms*

$$z_\ell + \mu u_c^2, \quad z_\ell + u_a^2 + \mu u_c^2, \quad z_\ell + u_b^2, \quad z_\ell + u_a^2 + u_a \cdot u_c + \mu u_c^2$$

with $\mu \in Z_2$.

Proof. Note that $H^2(\Gamma)$ is generated by z_ℓ, u_a^2, u_b^2, u_c^2, $u_a \cdot u_c$, $u_b \cdot u_c$. The proof is trivial if X is free of $u_a \cdot u_c$ and $u_b \cdot u_c$. Since $u_a^2 + u_b^2 = (u_a + u_b)^2 = u_a^2$ in $H^2(\langle a, ab\rangle)$, we can assume that X is free of $u_a^2 + u_b^2$. By Lemma 1.4, $\mathrm{Res}(\langle b, c \cdot a^{2^{\ell-2}}\rangle, \Gamma)$ $(z_\ell + u_b \cdot u_c) = 0$, so we also assume that X is free of $u_b \cdot u_c$. It remains only to consider the following cases:

(i) $X = z_\ell + u_a \cdot u_c$ or $z_\ell + u_b^2 + u_a \cdot u_c + u_c^2$, then X is reduced to $z_\ell + u_a^2 + u_a \cdot u_c$ or $z_\ell + u_a^2 + u_a \cdot u_c + u_c^2$ by the automorphism $\langle a, b, c\rangle \to \langle a, bc, c\rangle$.

(ii) $X = z_\ell + u_a \cdot u_c + u_c^2$, then X is reduced to $z_\ell + u_a^2 + u_a \cdot u_{c \cdot a^{2^{\ell-2}}}$ by $\langle a, b, c\rangle \to \langle a, abc \cdot a^{2^{\ell-2}}, c \cdot a^{2^{\ell-2}}\rangle$.

(iii) $X = z_\ell + u_b^2 + u_a \cdot u_c$, then X is reduced to $z_\ell + u_a^2 + u_a \cdot u_{c \cdot a^{2^{\ell-2}}} + u^2_{c \cdot a^{2^{\ell-2}}}$ by $\langle a, b, c\rangle \to \langle a, abc, c \cdot a^{2^{\ell-2}}\rangle$. The lemma is proved.

Let k be an integer and let $\{a_1, \ldots, a_{k-1}\}$ be a base of C_2^{k-1}. Set $\Psi_\ell = D(2^\ell) \times C_2^{k-1}$, then

$$H^*(\Psi_\ell) = P[u_a, u_b, u_1, \ldots, u_{k-1}, z_\ell]/(u_a^2 + u_a \cdot u_b).$$

We get

Lemma 1.6. *Let $X \in H^2(\Psi_\ell)$ with $\mathrm{Res}(\langle a^{2^{\ell-2}}\rangle, \Psi_\ell)X = u^2_{a^{2^{\ell-2}}}$. Then X can be reduced by an automorphism of Ψ_ℓ to one of the following forms*

$$z_\ell + \mu u_a^2 + \nu(1-\mu)u_b^2 + \sum_{i=1}^{m-1} u_{2i-1}\cdot u_{2i},$$

$$z_\ell + u_a^2 + u_a\cdot u_1 + \mu u_1^2 + \sum_{i=2}^{m-1} u_{2i-1}\cdot u_{2i},$$

$$z_\ell + u_a^2 + u_a\cdot u_1 + u_2^2 + \sum_{i=2}^{m-1} u_{2i-1}\cdot u_{2i},$$

with $\mu, \nu \in Z_2$.

Proof. By the proof of Lemma 1.5, we can suppose that $X = Y + \sum_{\substack{1\le i\le j\le k-1\\ j>1}} \mu_{ij}u_i\cdot u_j$ with Y being given in Lemma 1.5. If X is not free of $u_a\cdot u_1$ and $\mu_{1j}\neq 0$ for $j>1$, then $u_a\cdot u_1 + u_1\cdot u_j = u_1\cdot(u_a+u_j) = u_1\cdot u_j$ in $H^2(\langle a\cdot a_j, a_1, a_j\rangle)$, so we can assume that $\mu_{1j}=0$ for $j>1$. By appropriate changes of base of Ψ_ℓ, we can show that X is reduced to one of the required forms. For example, if $X = z_\ell + u_a^2 + u_a\cdot u_1 + u_3^2 + u_4^2 + u_3\cdot u_4$, then $X = z_\ell + u_{a.a_1}\cdot u_{a_1} + u^2_{b.a_4} + u_{a_3.a^{2^{\ell-2}}}\cdot u_{a_3\cdot a_4\cdot a^{2^{\ell-2}}}$ by the automorphism $\langle a,b,a_1,a_3,a_4\rangle \to \langle a\cdot a_1, b\cdot a_4, a_1, a_3\cdot a^{2^{\ell-2}}, a_3\cdot a_4\cdot a^{2^{\ell-2}}\rangle$. The lemma follows.

Since the factor sets of the central extensions

$$0\to Z_2\to G\to D(2^\ell)\to 1,$$

$$0\to Z_2\to H\to \Gamma\to 1$$

with $G = D(2^{\ell+1})$, $S(2^{\ell+1})$, $Q(2^{\ell+1})$ and $H = D^+(2^{\ell+2})$, $Q^+(2^{\ell+2})$ are respectively

$$z_\ell,\quad z_\ell + u_a^2,\quad z_\ell + u_b^2,\quad z_\ell + u_a^2 + u_a\cdot u_1,\quad z_\ell + u_a^2 + u_a\cdot u_1 + u_1^2$$

(see e.g. [6]), we have

Lemma 1.7. *Let $0\to Z_2\to G\to \Psi_{\ell+1}\to 1$ be a central extension with factor set $0\neq z\in H^2(\Psi_{\ell+1})$ having one of the forms given in Lemma 1.6, then G is isomorphic to one of the following groups*

$$M\cdot\underbrace{E\cdot\ldots\cdot E}_{m-1\ times}\times C_2^{k-2m+1},\quad N\cdot\underbrace{E\cdot\ldots\cdot E}_{m-2\ times}\times C_2^{k-2m+2},$$

$$D^+(2^{\ell+3})\cdot C_4\cdot \underbrace{E\cdot\ldots\cdot E}_{m-2\ times}\times C_2^{k-2m+1}$$

2. PROOFS OF THEOREMS A AND B

Our proofs are based on the following

Proposition 2.1. *Let (J) $0\to Z\to J\to K\to 1$ be a central extension with $Z\cong Z_p$ and with factor set $z\in H^2(K)$. Then $Z\subset\Phi(J)$ iff $z\neq 0$ in $H^2(K)$. Furthermore, if $z\neq 0$, then $\Phi(J)/Z=\Phi(K)$.*

Proof. It is obvious that $z=0$ implies that $J=K\times Z$, so $Z\not\subset\Phi(J)$. Conversely, assume that $Z\not\subset\Phi(J)$, then there exists a maximal subgroup H of J such that $Z\not\subset H$. Hence $J=H\cdot Z=H\times Z$, so the extension (J) splits. This implies $z=0$.

For $z\neq 0$, since $J/\Phi(J)$ is elementary abelian and $J/\Phi(J)\cong J/Z/\Phi(J)/Z=K/\Phi(J)/Z$, we have $\Phi(K)\subset\Phi(J)/Z$. On the other hand, let L be normal in J with $L/Z=\Phi(K)$, then $J/L\cong J/Z/L/Z=K/\Phi(K)$. Since $K/\Phi(K)$ is elementary abelian, we have $\Phi(J)\subset L$. Hence $\Phi(J)=L$, so $\Phi(J)/Z=\Phi(K)$.

With the assumptions of Proposition 2.1, let $\{E_r(J)\}$ be the Hochschild-Serre spectral sequence for the central extension (J). So $E_2(J)=H^*(K)\otimes H^*(Z)$. We suppose that $z\neq 0$ in $H^2(K)$. Following (1.1), set

$$H^*(Z)=\begin{cases} P[u;1], & \text{if } p=2\\ E[u;1]\otimes P[u;2], & \text{if } p>2.\end{cases}$$

Since $d_2(u)=z$, we have

$$\begin{aligned}E_3(J)=H^*(K)/(z)\otimes Z_p[\beta u]\\ \oplus Ann_{H^*(K)}(z)\otimes Z_p[\beta u]u\end{aligned}$$

(see e.g. [4] or [5]), and there is a bijection

$$H^2(J)\xrightarrow{\theta}E_\infty^{0,2}(J)\oplus E_3^{1,1}(J)\oplus E_3^{2,0}(J),\quad x\mapsto x+F^{i+1}H^2(J),$$

with i the degree of Hochschild-Serre filtration of x, $0\leq i\leq 2$.

We also have

Lemma 2.2. *Let (L) be a central extension $0\to Z_p\xrightarrow{i}L\xrightarrow{\pi}J\longrightarrow 1$ with factor set $0\neq z'\in H^2(J)$. Then* *a) Every extension of a subgroup of $\Phi(J) \cap Z(J)$ by iZ_p is contained in $Z(\Phi(L))$;*

b) If $\theta z' \in E_3^{1,1}(J)$, then $E_\infty^{0,2}(L) = 0$;

c) If $\theta z' \in E_3^{2,0}(J)$, then the extension of Z by iZ_p is $Z \times iZ_p$, and is a subgroup of $\Phi(L) \cap Z(L)$.

Proof.

a) Let $a \in \Phi(J) \cap Z(J)$ and $b \in L$ such that $\pi(b) = a$. For g, $h \in L$, since $[g, b]$ and $[h, b]$ belong to iZ_p, we have $[g^p, b] = 1$ and $[[g, h], b] = 1$ in iZ_p. a) is then proved.

b) If $\theta z' \in E_3^{1,1}(J)$, then $\theta z'$ is of form $x \otimes u$, with $x \in H^1(K)$. Let g be an element of K such that $x(g) \neq 0$ and (D) the central extension $0 \to Z_p \to D \to \langle g \rangle \to 1$. Then $\mathrm{Res}(D, J)z' \mapsto x \otimes u \in E_\infty^{1,1}(D)$. Hence $\beta z' \mapsto x \otimes \beta u \in E_\infty^{1,2}(D)$ which is non-zero. Since $x \otimes \beta u$ is not of form $y \cdot x \otimes u$ with $y \in H^1(\langle g \rangle)$, it follows that $\beta z' = d_3(\beta u) \neq 0$. This implies $E_\infty^{0,2}(J) = 0$.

c) Obvious from the definition of the Hochschild-Serre filtration on Bar cochains.

From Lemma 2.2 a) and c), we obtain

Lemma 2.3. *With the assumptions of Proposition 2.1 and Lemma 2.2, assume that $Z(\Phi(L))$ is cyclic, then $\Phi(J) \cap Z(J)$ is cyclic and $\theta z' \in E_\infty^{0,2}(J)$.*

Lemma 2.4. *With the assumptions of Proposition 2.1 and Lemma 2.2, assume that $\Phi(L) \cap Z(L)$ is cyclic and $E_\infty^{0,2}(L) \neq 0$, then $\Phi(J) \cap Z(J)$ is cyclic and $\theta z' \in E_\infty^{0,2}(J)$.*

Proof. Consider the extention (K), with Z an arbitrary subgroup of $\Phi(G) \cap Z(J)$ of order p. Since $\Phi(L) \cap Z(L)$ and $E_\infty^{0,2}(L) \neq 0$, it follows from Lemma 2.2 that $\theta z' \in E_\infty^{0,2}(J)$. So $\mathrm{Res}(Z, J)z' = \beta u$. By Lemma 1.2, $\Phi(J) \cap Z(J)$ is cyclic. The lemma follows.

Proof of Theorem A. Let $|G| = p^{n+\ell}$ and $|\Phi(G)| = p^\ell$. By Lemmas 2.3 and 2.4, we get a sequence of central extensions (G_i) $0 \to Z_p \to G_i \to G_{i+1} \to 1$, $1 \leq i \leq \ell$, with $G_1 = G$, $G_{\ell+1} = C_p^n$, and the factor set z_i of (G_i) satisfies $z_i \in E_\infty^{0,2}(G_{i+1})$, $\beta z_\ell = 0$. Hence $\Phi(G_\ell), \Phi(G_{\ell-1}), \ldots, \Phi(G_1)$ are cyclic. The theorem is proved.

In order to prove Theorem B, we need

Lemma 2.5. *If K is not elementary abelian, then $\Phi(J)$ is cyclic iff $\Phi(K)$ is cyclic and $z \mapsto \alpha v \in E_\infty^{0,2}(J)$ with $0 \neq \alpha \in Z_p$.*

Proof. By Proposition 2.1, we have the central extension $0 \to Z \to \Phi(J) \to \Phi(K) \to 1$. So $\Phi(J)$ is cyclic iff $\Phi(K)$ is cyclic and $\mathrm{Res}(\Phi(K), K)z \neq 0$. The lemma follows.

Lemma 2.6. *Let* $K = C_{p^\ell} \times C_p^{k-1}$ *or* $D(2^\ell) \times C_2^{k-1}$ *and* J *be one of the groups given in Lemmas 1.3 and 1.7. Then* $E_\infty^{0,2}(J) \neq 0$ *iff* $J = C_{p^{\ell+1}} \times C_p^{k-1}$ *or* $D(2^{\ell+1}) \times C_2^{k-1}$.

Proof. It is obvious that $E_\infty^{0,2}(J) \neq 0$ iff $\beta z = 0 \pmod z$ in $H^*(K)$. This fact is equivalent to $z = v_1$ or $z = Z_\ell$. The lemma follows.

Proof of Theorem B. We proceed by induction on ℓ. The theorem is clearly true for $\ell = 1$. Assume that it holds for $\ell - 1$ ($\ell \geq 2$). Let Z be the subgroup of $\Phi(G)$ of order p. By Lemmas 2.5 and 2.6, $\Phi(G/Z)$ is cyclic, $G/Z \cong C_{p^\ell} \times C_p^{n-1}$ or $D(2^\ell) \times C_2^{n-1}$ and the factor set for the central extension $1 \to Z \to G \to G/Z \to 1$ is of one of the forms given in Lemmas 1.2 and 1.6. The theorem follows from Lemmas 1.3 and 1.7.

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