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ON SOLUTION OF BOUNDARY-VALUE PROBLEMS

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FOR NONLINEAR HYPERBOLIC EQUATIONS WITH DISCONTINUOUS COEFFICIENTS

Department of Mathematics Received January 25, 1991

Hanoi Pedagogical Institute 1. Revised June 20, 1993

NGUYEN HUU CONG

Abstract. In this paper, we study a boundary-value problem for nonlinear partial differential equations of hyperbolic type with discontinuous coefficients. The existence and uniqueness of solution of the problem is proved. This paper can be considered as an extension of the results in [1], [2].

1 INTRODUCTION

In [1] we considered the following boundary-value problem (BVP):

$$A(t, x, y, D_t^i D_x^j D_y^\beta) U(t, x, y) = F(t, x, y),$$
(1.1)

$$D_t^i U(t, x, y) \big|_{t=0} = \psi_i(x, y), \quad i = 1, 2, \dots, m,$$
 (1.2a)

$$\alpha'_{ij\beta}D_t^i D_x^j D_y^{\beta} U(t, x, y)\big|_{x=-0} = \alpha''_{ij\beta}D_t^i D_x^j D_y^{\beta} U(t, x, y)\big|_{x=+0},$$
 (1.3)

with
$$|i+j+|\beta| = m, \quad j = 1, 2, ..., 2\left[\frac{m+1}{2}\right] - 1$$

where $(t, x, y) \in G := (0, T) \times \Omega \subset \mathbb{R}^{n-1}$,

$$\begin{split} D_t^i &:= \frac{\partial^i}{\partial t^i} \;, \quad D_x^j := \frac{\partial^j}{\partial x^j} \;, \quad D_y^\beta := D_{y_1}^{\beta_1} D_{y_2}^{\beta_2} \dots D_{y_{n-2}}^{\beta_{n-2}} \quad \text{and} \\ A(t, x, y, D_t^i D_x^j D_y^\beta) &:= \sum_{i+j+|\beta| \leq m+1} a_{ij\beta(t, x, y)} D_t^i D_x^j D_y^\beta \end{split}$$

is a partial differential operator of hyperbolic type with respect to t whose coefficients are bounded for $i+j+|\beta|\leq m+1$, piecewise sufficiently smooth for

 $i+j+|\beta|=m+1$, and $F(t,x,y)\in L^2(G)$. The coefficients $a_{ij}_{\beta(t,x,y)}$ (with $i+j+|\beta|=m+1$) are supposed to possess discontinuous points only on hyperplane $\{x=0\}$, tend to 0 while x tends to ± 0 for the odd j and satisfy the following conditions:

conditions: $\frac{a_{ij\beta}''}{a_{ij\beta}'} = \frac{a_{(m+1)00}''}{a_{(m+1)00}'} \left\{ \frac{a_{i2[(m+1)/2]\beta'}'}{a_{i2[(m+1)/2]\beta'}'} \right\}^{j/2[(m+1)/2]}$ (1.4)

for the even $j=0,2,4,\dots 2[(m+1)/2]$ where $a_{ij\beta}''=\lim a_{ij\beta(t,x,y)}$ and $a_{ij\beta}'=\lim a_{ij\beta(t,x,y)}$ while x tends to ± 0 respectively. In the case when m is even we require that $a_{1m0}=a_{0m1}$. The coefficients $\alpha_{ij\beta}'$, $\alpha_{ij\beta}''$ in the condition (1.3) satisfy the relations below:

$$\frac{\alpha''_{ij\beta}}{\alpha'_{ij\beta}} = \left\{ \frac{a''_{(m+1)00}}{a'_{(m+1)00}} \right\}^{(-1)^{ij}} \left\{ \frac{\alpha''_{m00}}{\alpha'_{m00}} \right\}^{(-1)^{j}} \frac{a''_{i2[(j+1)/2]\beta}}{a'_{i2[(j+1)/2]\beta}}$$
(1.5)

In the particular case when the initial conditions (1.2a) is replaced by

$$D_t^i U(t, x, y) \big|_{t=0} = 0 \quad i = 1, 2, \dots, m$$
 (1.2b)

we proved the following energy inequality:

$$||U(t,x,y)||_{H^{m}(G)} \le C ||AU(t,x,y)||_{L^{2}(G)}.$$
 (1.6)

In [2] for the problem (1.1), (1.2b), (1.3) (with conditions (1.4), (1.5)) we also proved the dual inequality. The energy inequality (1.6) and dual inequality ensure the existence and the uniqueness of solution of this problem for any right hand side $F(t,x,y) \in L^2(G)$ (for details about this linear problem we refer to [1], [2]).

exists to C (0.1) such that the problem (2.2), (1.26), (1.3) possesses a assistion Uo, then the ROOITAUPA RABORI SON HOROITAUPA RABORINON ROF MELON-YRAGUNOR SON for

Lemma 2.1. Assume the conditions of Theorem 2.1 and let suppose that there

In this section we consider a new problem by replacing equation (1.1) in the above problem (1.1), (1.2b), (1.3) (with the conditions (1.4), (1.5)) by the nonlinear equation below:

$$A(t, x, y, D_t^i D_x^j D_y^\beta) U(t, x, y) = F(t, x, y, D_t^i D_x^j D_y^\beta U).$$
 (2.1)

For this nonlinear equation, the following theorem holds.

the relations below:

Theorem 2.1. If $F(t,x,y,D_t^iD_x^jD_y^\beta U)$ is a real nonlinear bounded function of variables t, x, y, $D_t^i D_x^j D_y^\beta U$ (with $i+j+|\beta| \leq m-1$) and $F(t,x,y,D_t^i D_x^j D_y^\beta U)$ has first and second bounded derivatives with respect to variable $D_t^i D_x^j D_y^\beta U$ in G_i then the solution of the BVP (2.1), (1.2b), (1.3) (with the conditions (1.4), (1.5)) exists and is unique.

Proof of Theorem 2.1. For each $\tau \in [0,1)$ let us define the differential equation

$$\Phi_{\tau}(t,x,y,U) = A(t,x,y,D_t^iD_x^jD_y^\beta)U(t,x,y) - \tau F(t,x,y,D_t^iD_x^jD_y^\beta U) = 0. \quad (2.2)^{\frac{1}{2}}$$
This is a factor of the contract of

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Consider the iterated Newton's type methods

$$\left[A - \tau \left(\sum_{i+j+|\beta| \le m-1} F'_{ij\beta}(U_k)\right) D_t^i D_x^j D_y^\beta\right] (U_{k+1} - U_k) = -\Phi_\tau(U_k), \qquad (2.3a)$$

or
$$\left[A - \tau \left(\sum_{i+j+|\beta| \le m-1} F'_{ij\beta}(U_k) \right) D_t^i D_x^j D_y^\beta \right] (U_{k+1} - U_k) = -[AU_k - \tau F(U_k)], \quad (2.3b)$$

$$F'_{ijeta}(U_k) := rac{\partial F}{\partial D^i_i D^j_x D^eta_y U}(U_k).$$
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The relation (2.3b) is equivalent to

$$\begin{bmatrix} A - \tau \Big(\sum_{i+j+|\beta| \le m-1}^{F'_{ij\beta}} (U_k) \Big) D_t^i D_x^j D_y^\beta \Big] U_{k+1}$$

$$= w \ ((31), (41) \text{ snotthere disk} = \tau F(U_k) + \tau \Big(\sum_{i+j+|\beta| \le m-1}^{(42,1)} F'_{ij\beta}(U_k) \Big) D_t^i D_x^j D_y^\beta U_{k+1} \text{ beyon} (2.3c)$$
where the existence and the unique $m-1$ and the unique $m-1$ are the existence and the unique $m-1$ and $m-1$ are the existence and the unique $m-1$ and $m-1$ are the existence and the unique $m-1$ and $m-1$ are the existence and the unique $m-1$ and $m-1$ are the existence and the unique $m-1$ and $m-1$ are the existence and the unique $m-1$ and $m-1$ are the existence and the unique $m-1$ and $m-1$ are the existence and the unique $m-1$ are the existence and the unique $m-1$ are the existence and the unique $m-1$ and $m-1$ are the existence and the unique $m-1$ are the existence and the unique $m-1$ are the existence and the unique $m-1$ and $m-1$ are the existence are the existence and $m-1$ are the existence $m-1$ are the existence $m-1$ and $m-1$ are the existence $m-1$ are the existence $m-1$ are the existence $m-1$ and $m-1$ are the existence $m-1$ are the existence $m-1$ are the existence $m-1$ are the existence $m-1$ and $m-1$ are the existence $m-1$ are the existence $m-1$ and $m-1$ are the existence $m-1$ and $m-1$ are the existence $m-1$ are the existence $m-1$ and $m-1$ are the existence $m-1$ are the existence $m-1$ and $m-1$ are the existenc

For completing the proof of Theorem 2.1, we need the following lemma. The bush

Lemma 2.1. Assume the conditions of Theorem 2.1 and let suppose that there exists $\tau_0 \in [0,1)$ such that the problem (2.2), (1.2b), (1.3) possesses a solution U_0 , then the equation $\Phi_{\tau}(t,x,y,U) = 0$ possesses an unique solution in $H^{m}(G)$ for some $\tau \in (\tau_0, 1)$.

In this section we consider a new problem by replacing equation (1.1) in

Proof of Lemma 2.1. Replacing k by k-1 in (2.3c) we get

Substituting AUk from (2.4) in (2.3b) yields and a notional that that the bayed at at

By setting $W_k = U_{k+1} + U_k$ and $L_k = \left[A - \tau \left(\sum_{i+j+|\beta| \le m-1} F'_{ij\beta}(U_k)\right) D_t^i D_x^j D_y^\beta\right]$, the relation (2.5) takes the form:

 $\sum_{k=1}^{N_{guyen}} F_{ij\beta}(U_0) D_i^* D_k^* U_0 = -(AU_{guyen} F_{ij\beta}(U_0)) D_i^* D_k^* D_y^* U_0 = -(AU_{guyen} F_{ij\beta}(U_0)) D_i^* D_k^* U_0 = -(AU_{guyen} F_{ij\beta}(U_0)) D_i^* U_0 = -(AU_{guyen} F_{ij\beta$

 $g_k = F(U_k) - F(U_{k-1}) - \Big(\sum_{i+j+|\beta| \le m-1} F'_{ij\beta}(U_{k-1})\Big) D_t^i D_x^j D_y^\beta (U_k - U_{k-1}).$

It is evident that the differential operator L_k defined as above is a linear operator of hyperbolic type whose coefficients satisfy all the conditions as required for the coefficients of A (in particular the conditions (1.4), (1.5)). Therefore the results reported in Section 1 for the problem (1.1), (1.2b), (1.3) can be apply to the problem (2.6), (1.2b), (1.3). Using the energy inequality (1.6), we obtain

$$\|W_k\|_{H^m(G)} \le \tau C \|g_k\|_{L^2(G)}$$
. (2.7) From the inequalities (2.1) and (2.9) it follows

Since

$$\|g_{k}\|_{L^{2}(G)} = \|F(U_{k}) - F(U_{k-1})\|_{L^{2}(G)} + \|F(U_{k-1}) - F(U_{k-1})\|_{L^{2}(G)} + \|F(U_{k-1})$$

The inequalities (2.12) and (2.13) prove inequality word (2.12) and (2.12)

(Here we recall that the function F and its first and second derivatives are supposed to be bounded).

(We rewrite (2.8) in another form: (1.0) (1.0)

$$\frac{\Gamma \tau C}{2} \|W_k\|_{H^m(G)} \le \left[\frac{\Gamma \tau C}{2} \|W_{k-1}\|_{H^m(G)} \right]^2. \tag{2.8b}$$

Thus

$$\left\| \frac{\Gamma \tau C}{2} \| W_n \|_{H^m(G)} \le \left\| \frac{\Gamma \tau C}{2} \| W_0 \|_{H^m(G)} \right\|^{2^n}$$

Now we estimate $||W_0||_{H^m(G)}$. In the equality (2.3b) by setting k=0, we get

$$\left[A - \tau \left(\sum_{i+j+|\beta| \le m-1} F'_{ij\beta}(U_0)\right) D_t^i D_x^j D_y^\beta \right] (U_1 - U_0) = -[AU_0 - \tau F(U_0)]. \quad (2.3d)$$

 $g_k = F(U_k) - F(U_{k-1}) - \left(\sum_{i,j} F_{ij\beta}(U_{k-1})\right) D_i^* D_2^*$ shows and in the second of the second sec

$$\|W_0\|_{H^m(G)} \leq C \|AU_0 - \tau F(U_0)\|_{L^2(G)}$$
 rotaredo resulta si evode $C \|AU_0 - \tau_0 F(U_0) + (\tau_0 - \tau) F(U_0)\|_{L^2(G)}^{\frac{1}{2}}$, then the property of the description and the visits satisfication and the property of the property o

coefficients of A (in particular the conditions (0.1, 0.1)). Therefore the results and hence, since $U_0 = 0$, we have problem 1 for the problem with $U_0 = 0$.

problem (2.6), (1.2b), (1.3) Using the energy inequality (1.6), we obtain
$$\|W_0\|_{H^m(G)} \leq C(\tau - \tau_0) \|F(U_0)\|_{L^2(G)} .$$
 (2.11)

From the inequalities (2.11) and (2.9) it follows

$$\frac{\Gamma \tau C}{2} \|W_n\|_{H^m(G)} \leq \left[\frac{\Gamma C^2}{2} (\tau - \tau_0) \|F(U_0)\|_{L^2(G)} \right]^{2^n}. \tag{2.12}$$

By choosing τ closed enough to τ_0 one have

$$\frac{\Gamma C^2}{2} (\tau - \tau_0) \|F(U_0)\|_{L^2(G)} \leq 1.$$
 (2.13)

The inequalities (2.12) and (2.13) prove Lemma 2.1.

Finally, observing that the problems (2.2), (1.2b), (1.3) possesses the solution for $\tau = 0$ (see [1], [2]) and applying parameter continuation method, Theorem 2.1 easily follows. \Box

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Faculty of Mathematics,
Mechanics and Informatics
Hanoi University
Nguyen Trai Str. 90, Dong Da

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Abstract. The main purpose of this paper is to finend some properties of mide monogenic functions, which are a generalization of monogenic functions in higher dimensions, for a class of functions satisfiend Velva tips generalized Creshly Righers equations in Chifford analysis. It is proved that the March Decreas is cold for these functions.

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INTRODUCTION. I A to A so made

In [1] the theory of functions taking values in a Clifford algebra was studied. It is proved that many important properties of holomorphic functions of one complex variable may be extended homeonogenic functions which are solution of the generalized Cauchy-Riemann equations and playion important role in theoretical physics.

Following this way, in [6] Le Hung Son introduced a version of multi-mone genic functions which are a generalleation of monogenic functions in higher dimensions and proved some properties of these functions; among their there are the Hartogs extension theorems

The purpose of this paper is to extend these results for functions satisfying Vekua-type generalized Cauchy-Riemann equations, which are a generalization of multi-monogenic functions for Vekua-type in Clifford analysis. It is proved that the Hartogs extension shooted is valid for these functions. This is a generalization of some results in [4], [5], [6], [7].