THE STRUCTURE OF SPECIAL CLASSES OF GRAPHS AND APPLICATION

Abstract

Let $A \subset \mathbb{F}_q$, a finite field with q elements, denote $\mathcal{V}_n(A) = (A-A)\cdot(A-A)\cdots(A-A)$, where the product is taken n times. D. Hart, A. Iosevich, J. Solymosi proved that if $|A| \geq Cq^{\frac{1}{2}+\frac{1}{2n}}$, with a sufficiently large absolute constant C, then

$$\mathcal{V}_n(A) = \mathbb{F}_q,$$

where the product is taken n times. A. Balog also obtain the following result, if $|A| \geq C.q^{\frac{1}{2}+\frac{1}{2^k}}$, then

$$\mathcal{V}_{2k+1}(A) = \mathbb{F}_q.$$

Using the graph theoretic method, we obtain the following improvement, if $|A| \geq C.q^{\frac{1}{2}+\frac{1}{3/2\cdot 2^k}}$, then

$$\mathcal{V}_{2k+1}(A) = \mathbb{F}_q.$$

We also obtain similar results in the setting of finite cyclic rings.