Spanning Graph Designs

Abstract:

Balanced Incomplete Block Designs, BIBD, have been introduced a long time ago in statistics. The "simplest" examples are the Steiner Triple Systems: $S(n, 3, 1)$. This means a collection of triple from a set V of cardinality n such that every pair $\{x, y\} \in V$ belongs to exactly one triple. There are obvious arithmetic conditions on n and they are well known to be also sufficient.
There are obvious extensions: $S(n, k, 1)$, $\quad , S(n, k, \lambda)$. One of the most remarkable results in combinatorial design theory is Richard Wilson's theorem that if n is large enough and satisfies the obvious arithmetic conditions then $S(n, k, \lambda)$ exists.
A recent remarkable result was recently proved by P. Keevash: $S(n, k, m, \lambda)$ exists for all n large enough when the arithmetic conditions are satisfied (here m stands for m-tuples, that is we want a collection of k subsets such that every m subset appears in $\lambda \quad k$ subsets.
Yet another generalization are the graph designs $S(n, G, \lambda)$. In this case we wish to partition the edges of K_n into subgraphs isomorphic to a given graph G so that every edge appears in exactly λ distinct copies.
When $\parallel G \parallel = n$ we call $S(n, G, \lambda)$ a spanning graph design.
For which cubic graphs G there is a cubic graph design $S(n, G, 1)$?
I believe that if n is large enough they always exist.