

# Optimality conditions based on the Fréchet second-order subdifferential

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**Abstract:** This paper focuses on second-order necessary optimality conditions for constrained optimization problems on Banach spaces. For problems in the classical setting, where the objective function is  $C^2$ -smooth, we show that strengthened second-order necessary optimality conditions are valid if the constraint set is generalized polyhedral convex. For problems in a new setting, where the objective function is just assumed to be  $C^1$ -smooth and the constraint set is generalized polyhedral convex, we establish sharp second-order necessary optimality conditions based on the Fréchet second-order subdifferential of the objective function and the second-order tangent set of the constraint set. Three examples are given to show that the used hypotheses are essential for the new theorems. Our second-order necessary optimality conditions refine and extend several existing results.

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