LOCAL MINIMIZERS OF SEMI-ALGEBRAIC FUNCTIONS

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ABSTRACT. Consider a semi-algebraic function $f\colon \mathbb{R}^n \to \mathbb{R}$, which is continuous around a point $\bar{x} \in \mathbb{R}^n$. Using the so–called *tangency variety* of f at $\bar{x}$, we first provide necessary and sufficient conditions for $\bar{x}$ to be a local minimizer of f, and then in the case where $\bar{x}$ is an isolated local minimizer of f, we define a "tangency exponent" $\alpha_* > 0$ so that for any $\alpha \in \mathbb{R}$ the following four conditions are always equivalent:

(i) the inequality $\alpha \geq \alpha_*$ holds;
(ii) the point $\bar{x}$ is an α-order sharp local minimizer of f;
(iii) the limiting subdifferential ∂f of f is $(\alpha - 1)$-order strongly metrically subregular at $\bar{x}$ for 0; and
(iv) the function f satisfies the Łojaseiwcz gradient inequality at $\bar{x}$ with the exponent $1 - \frac{1}{\alpha}$.

Besides, we also present a counterexample to a conjecture posed by Drusvyatskiy and Ioffe in [Math. Program. Ser. A, 153(2):635–653, 2015].

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