Coderivatives and Implicit Multifunction Theorems

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Abstract

The dissertation has four chapters. The first chapter introduces several concepts and tools from Set-Valued Analysis and Variational Analysis. In the second chapter, we investigate the local Lipschitz-like property and the Robinson stability of the solution map of a parametric generalized linear constraint system (GLCS) by means of normal coderivative, the Mordukhovich criterion, and a related theorem due to Levy and Mordukhovich [Math. Program., 99 (2004), pp. 311–327]. Among other things, the obtained results yield uniform local error bounds and traditional local error bounds for the linear complementarity problem and the general affine variational inequality problem, as well as verifiable sufficient conditions for the Lipschitz-like property of the solution map of the linear complementarity problem and a class of affine variational inequalities, where all components of the problem data are subject to perturbations. The third chapter shows analogues of the results of the previous chapter for the case when the GLCS undergoes linear perturbations. In the final chapter, we analyze the stability of the stationary point set map of a C^2-smooth parametric optimization problem with one C^2-smooth functional constraint under total perturbations by applying some theorems of Levy and Mordukhovich [*Math. Program.,* 99 (2004), pp. 311–327] and several related results. We not only show necessary and sufficient conditions for the local Lipschitz-like property of the stationary point set map, but also sufficient conditions for its Robinson stability. These results lead us to new insights into the preceding deep investigations of Levy and Mordukhovich in the above-cited paper and of Qui [*J. Optim. Theory Appl.,* 161 (2014), pp. 398–429; *J. Glob. Optim.,* 65 (2016), pp. 615–635], and allow us to revisit and extend several stability theorems in indefinite quadratic programming.

Keywords and Phrases: Generalized linear constraint system, Linear complementarity problem, Affine variational inequality, Solution map, Coderivative, Lipschitz-like property, Robinson stability, Metric regularity, Uniform local error bound, Smooth parametric optimization problem, Stationary point set map.

Related Topics: Set-Valued Analysis, Variational Analysis, Optimization Theory.