GENERIC PROPERTIES FOR SEMIALGEBRAIC PROGRAMS

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Abstract. In this talk, we study genericity for the following parameterized class of nonlinear programs:

$$\text{minimize } f_u(x) := f(x) - \langle u, x\rangle \quad \text{subject to} \quad x \in S,$$

where $f\colon \mathbb{R}^n \to \mathbb{R}$ is a polynomial function and $S \subset \mathbb{R}^n$ is a closed semi-algebraic set, which is not necessarily compact. Assume that the constraint set S is regular. It is shown that there exists an open and dense semi-algebraic set $\mathscr{U} \subset \mathbb{R}^n$ such that for any $\bar{u} \in \mathscr{U}$, the restriction of $f_{\bar{u}}$ on S is good at infinity and if $f_{\bar{u}}$ is bounded from below on S, then for all vectors $u \in \mathbb{R}^n$, sufficiently close to $\bar{u}$, the optimization problem $\min_{x\in S} f_u(x)$ has the following properties: the objective function f_u is coercive and has the same growth at infinity on the constraint set S, there is a unique optimal solution, lying on a unique active manifold, at which the strong second-order sufficient conditions, the quadratic growth condition and the global sharp minima hold, and all minimizing sequences converge. Furthermore, the active manifold is constant, and the optimal solution and the optimal value function depend analytically under local perturbations of the objective function. As a consequence, for almost all polynomial optimization problems, we can find a natural sequence of computationally feasible semidefinite programs, whose solutions give rise to a sequence of points in $\mathbb{R}^n$ converging *finitely* to the optimal solution of the original problem.

This result is joint work with Gue Myung Lee.

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Date: March 21, 2016.