Besov Space Regularity Conditions for Weak Solutions of the Navier-Stokes Equations

Abstract: Consider a bounded domain Ω in $\mathbb{R}^3$ with smooth boundary and a weak solution u of the Navier-Stokes system in $\Omega \times [0,T]$. Our aim is to develop regularity and uniqueness conditions for u which are based on the Besov space

$$B^{q,s}(\Omega) := \left\{ v \in L^2_\sigma(\Omega) : \|v\|_{B^{q,s}(\Omega)} := \left(\int_0^\infty \|e^{-\tau \, A_v}\|_q^s < \infty \right)^{\frac{1}{s}} \right\}.$$

with $2 < s < \infty, 3 < q < \infty, \frac{2}{s} + \frac{3}{q} = 1$; here A denotes the Stokes operator. Our main results on the regularity of u exploits a variant of the space $B^{q,s}(\Omega)$ in which the integral in time has to be considered only on finite intervals $(0, \delta)$.