#### Seminar khoa học

### Một ngày với Tối ưu và Tính toán khoa học

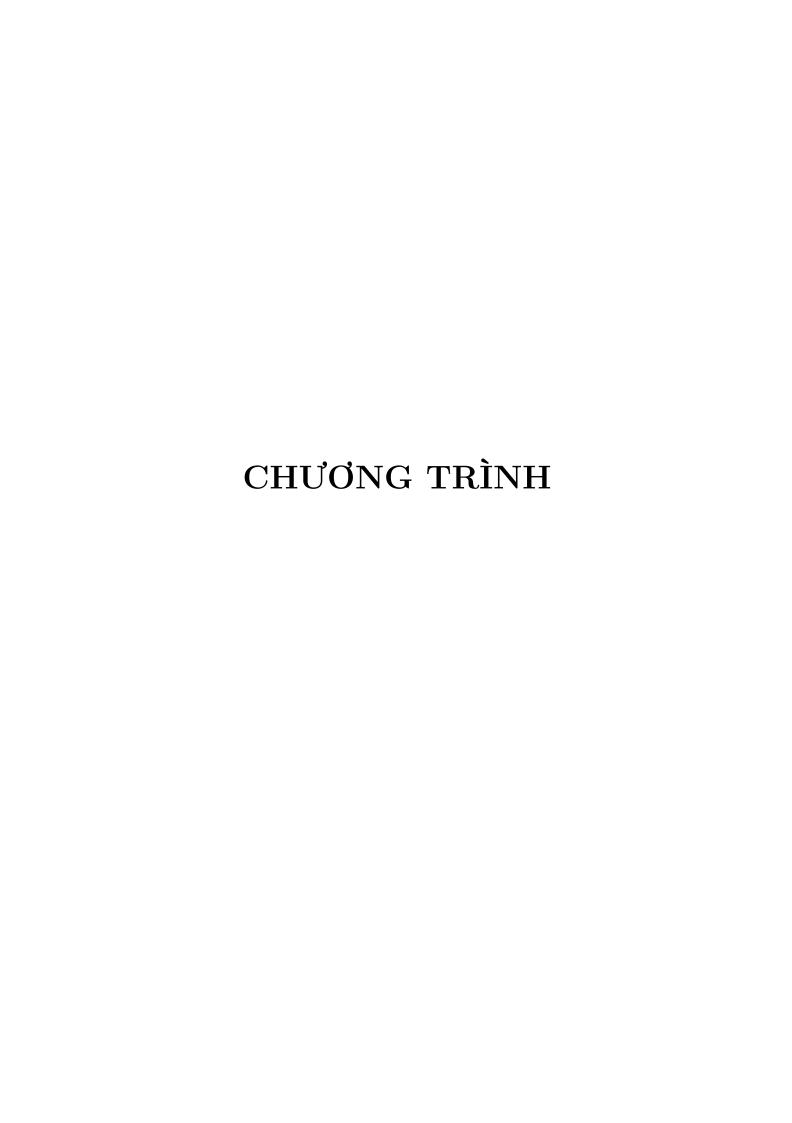
**Thời gian:** ngày 26 tháng 11 năm 2025 (thứ Tư)

Địa điểm: Phòng 508, nhà A6, Viện Toán học

Cơ quan tổ chức: Viện Toán học (phối hợp giữa Trung tâm Nghiên cứu và Đào tạo

Toán học Quốc tế với Phòng Tối ưu và Tính toán khoa học)

Đơn vị tài trợ: Trung tâm Nghiên cứu và Đào tạo Toán học Quốc tế - Viện Toán học



08:50 - 09:00 Khai mạc

#### Chủ tọa: **Bùi Trọng Kiên**

- 09:00 09:40 **Nguyễn Thị Vân Hằng** (Viện Toán học)

  Convergence of augmented Lagrangian methods for composite optimization problems
- 09:40 10:20 **Phạm Thị Hoài** (Đại học Bách khoa Hà Nội)

  Composite optimization models via proximal gradient method with a novel enhanced adaptive stepsizes
- 10:20 11:00 **Lê Hải Yến** (Viện Toán học) Shrinking-cutting projection algorithms for quasiconvex equilibrium problems

#### Chủ toa: **Bùi Trọng Kiên**

- 13:30 14:10 **Nguyễn Ngọc Luân** (Trường Đại học Sư phạm Hà Nội)

  Strong duality and solution existence under minimal assumptions in conic linear programming
- 14:10 14:50 **Nguyễn Kiều Linh**(Học viện Công nghệ Bưu chính Viễn thông)

  Inner δ-approximation of the convex hull of finite sets in two-dimensional normed spaces
- 14:50 15:30 **Lê Xuân Thanh** (Viện Toán học)

  Scheduling sport tournaments with two court types
- 15:30 15:50 **Tiệc trà**

#### Chủ toa: Lê Xuân Thanh

- 15:50 16:30 **Nguyễn Thị Thu Thủy** (Đại học Bách khoa Hà Nội)

  A strongly convergent algorithm for bilevel split variational inequalities with quasimonotone operators
- 16:30 17:10 **Trần Sỹ Toàn** (Đại học Bách khoa Hà Nội)

  An alternated inertial algorithm for solving a class of split variational inequality problems
- 17:10 17:50 **Nguyễn Năng Thiều** (Viện Toán học)

  Solving a class of nonconvex quadratic programs by inertial

  DC algorithms

# TÓM TẮT BÁO CÁO

## Convergence of augmented Lagrangian methods for composite optimization problems

Nguyễn Thị Vân Hằng\*, Ebrahim Sarabi

Abstract: This talk discusses local convergence analysis of the augmented Lagrangian method (ALM) for classes of composite optimization problems with nonunique Lagrange multipliers. We present and study new second-order variational properties, in terms of second subderivatives of functions and graphical derivatives of their subgradient mappings, for numerous important classes of functions in constrained and composite optimization problems. Employing these conditions and a second-order sufficient condition, we are able to establish Q-linear convergence of the primal-dual sequence for an inexact version of the ALM for composite programs.

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# Composite optimization models via proximal gradient method with a novel enhanced adaptive stepsizes

Phạm Thị Hoài\*, Nguyễn Phạm Duy Thái

Abstract: We first consider the convex composite optimization models with the local Lipschitzness condition imposed on the gradient of the differentiable term. The classical proximal gradient method will be studied with our novel enhanced adaptive stepsize selection. To obtain the convergence of the proposed algorithm, we establish a sufficient decrease type inequality associated with our new stepsize choice. This allows us to demonstrate the descent of the objective value from some fixed iteration and yield the sublinear convergence rate of the new method. Especially, in the case of locally strong convexity of the smooth term, our algorithm converges Q-linearly. We also further show that our method can be applied to nonconvex composite optimization problems provided that the differentiable function has a globally Lipschitz gradient. Finally, the efficiency of our proposed algorithms is shown by numerical results for numerous applicable test instances in comparison with the other state-of-the-art algorithms.

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## Inner $\delta$ -approximation of the convex hull of finite sets in two-dimensional normed spaces

Hoàng Nam Dũng, Nguyễn Kiều Linh\*, Hoàng Xuân Phú

**Abstract:** For a given finite set X and an approximation parameter  $\delta > 0$ , a convex polygon or polyhedron  $\mathcal{P}^{\text{inner}}$  is called an inner  $\delta$ -approximation of the convex hull conv X of X if convX contains  $\mathcal{P}^{\text{inner}}$  and the Hausdorff distance between them is not greater than  $\delta$ . In this talk, an algorithm for computing  $\mathcal{P}^{\text{inner}}$  in 2D is presented. This approximation approach can reduce the computation time. For example, if X consists of 1,000,000 random points in an ellipse, the computation time can be reduced by 11.20% if one chooses  $\delta$  to be equal to  $10^{-4}$  multiplied by the diameter of this ellipse. By choosing  $\delta = 0$ , our algorithms can be applied to very efficiently determine the exact convex hull conv X. Numerical experiments confirm that their time complexity is linear in n if X consists of n random points in an ellipse or a rectangle. Compared to others, our algorithm is much faster than the Quickhull algorithm implemented in the Qhull library, which in turn is faster than all currently available 2D convex hull functions in the Computational Geometry Algorithm Library (CGAL). If X consists of n = 100,000 random points in an ellipse or a rectangle, then our algorithm is 5.17 or 18.26 times faster than Qhull, respectively. This means that our algorithms run much faster for random points in polygonal frame figures such as rectangles than for random points in round and absolutely smooth frame figures such as ellipses. Moreover, the larger the point number n is, the higher the speedup factors of our algorithms are. For example, if X consists of n = 46,200,000 random points in an ellipse or a rectangle, then the speedup factors of our algorithm compared to Qhull are 8.46 and 22.44, respectively.

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## Strong duality and solution existence under minimal assumptions in conic linear programming

Nguyễn Ngọc Luân\*, Nguyễn Đông Yên

**Abstract:** Conic linear programs in locally convex Hausdorff topological vector spaces are addressed in this paper. Solution existence for the dual problem, as well as solution existence for the primal problem, and strong duality, are proved under minimal regularity assumptions. Namely, to get the results and a Farkas-type theorem for infinite-dimensional conic-linear inequalities, we employ the generalized Slater condition either for the primal problem or for the dual problem, as well as proper separation and the concept of quasi-regularity of convex sets. Illustrative examples are presented.

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#### Scheduling sport tournaments with two court types

Sigrid Knust, Melissa Koch, Lê Xuân Thanh\*

**Abstract:** In this talk, we propose a new variant of a sports tournament scheduling problem where additionally courts of two different types are considered. The goal is to find a schedule for a single round robin tournament where the total number of consecutive matches on the same court type for the players is minimized. We propose efficient construction methods to obtain optimal solutions for different cases.

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## Solving a class of nonconvex quadratic programs by inertial DC algorithms

Trần Hùng Cường, Yong<br/>do Lim, Nguyễn Năng Thiều $^*$ , Nguyễn Đông Yên

Abstract: In this talk, we present two inertial DC algorithms for indefinite quadratic programs under linear constraints (IQPs), where the constraint set may be unbounded. Using a qualification condition involving the normal cones of unbounded pseudo-faces of the polyhedral convex constraint set, the recession cones of the corresponding faces, and the quadratic form defining the objective function, we show that the resulting iteration sequences are bounded whenever the given IQP has a finite optimal value. Any cluster point of such a sequence is a Karush-Kuhn-Tucker point. Moreover, all cluster points of a given iteration sequence belong to a single connected component of the Karush-Kuhn-Tucker point set.

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#### A strongly convergent algorithm for bilevel split variational inequalities with quasimonotone operators

Nguyễn Thị Thu Thủy\*, Nguyễn Trung Nghĩa

Abstract: We investigate a class of bilevel variational inequality problems, focusing on those defined over the solution sets of split variational inequality problems with multiple output sets and quasimonotone cost operators. To solve this class of problems, we propose a strongly convergent algorithm in real Hilbert spaces that incorporates alternated inertial extrapolation with a self-adaptive step-size strategy. The proposed method unifies and extends several existing frameworks, offering two key advantages: (1) it eliminates the need for line search or prior knowledge of problem-specific parameters such as the norm of transfer operators, Lipschitz constants, or strong monotonicity coefficients; and (2) it is applicable to quasimonotone operators, which encompass both pseudomonotone and monotone mappings as special cases. We validate the practical effectiveness of our algorithm through applications to urban traffic network equilibrium problems with quasimonotone cost structures, using multi-city network models. Numerical experiments demonstrate the robustness and computational efficiency of the proposed method, showing competitive or superior performance compared to state-of-the-art techniques in computing equilibrium flows in complex urban networks.

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## An alternated inertial algorithm for solving a class of split variational inequality problems

Nguyễn Thị Thu Thủy, Trần Sỹ Toàn\*

Abstract: In this talk, we propose an alternated inertial iterative algorithm with a self-adaptive step size for approximating solutions to a class of split variational inequality problems in real Hilbert spaces. The strong convergence of the generated sequence is established under the assumption that the involved operators are quasimonotone, a condition weaker than pseudomonotonicity or monotonicity, and without requiring prior knowledge of the norms of the transfer operators or the Lipschitz constants of the cost mappings. Furthermore, the allowable range of step sizes is broadened by relaxing constraints on several control parameters. Finally, the proposed algorithm is applied to a class of optimal control problems governed by variational inequalities, and numerical experiments are presented to demonstrate its effectiveness and implementability.

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### Shrinking-cutting projection algorithms for quasiconvex equilibrium problems

Lê Hải Yến\*, Lê Dũng Mưu

Abstract: In this talk, we propose three algorithms for solving Minty (dual) quasiconvex equilibrium problems by using the fact that its solution-set of such a problem is just the intersection onto the feasible domain of convex sets defined by the lower lever sets defined by the bi-function involved. These algorithms overcome the difficulty arising from the fact that the sum of a quasiconvex and a strongly convex function is not strongly convex, even not quasiconvex. The first algorithm is a shrinking one combining with a cutting hyper-plane to shrink the research domain. A main drawback of this algorithm is that, at each iteration k, it requires computing the projection onto the intersection of the feasible domain and k halfspaces. The second and third algorithms are modified versions of the first one, where the projection is just onto the feasible domain and one halfspace. Neither monotonicity property nor Lipschitz-type condition are required. Some computational results computed on randomly generated data showing the behavior of each proposed algorithm.

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