

Seminar: l -adic cohomology and Deligne's proof of Weil conjecture

The aim of this seminar is to prepare some basic materials to follow the lecture of Prof. Deninger. These include: l -adic cohomology, Picard-Lefschetz theory for algebraic varieties and application to Deligne's proof of Weil's conjecture.

Schedule: February 29 – March 2, 2024

Talk 1: Introduction to étale cohomology 1. This talk should serve as a quick introduction to étale cohomology, covering basic notions such as:

- étale topology, étale site and sheaves, locally constant sheaves and constructible sheaves and their relationship with the fundamental group,
- direct and inverse image, the existence of injective,
- definition and basic properties of étale cohomology such as the proper and smooth base change theorem for étale cohomology.

Ref: These materials can be found in Milne's lecture [Mi], chapters 4-10, 17, 20. Another source is Freitag-Kiehl's book [FK88, Chapter I], see also [Stacks, Chap. 59] and SGA 4.5
Speaker: Nguyễn Khánh Hưng (Univ. Of Paris Saclay)

Talk 2: Introduction to étale cohomology 2. This is a continuation of Talk 1 and should address the following topics:

- the comparison theorem for complex varieties
- the finiteness theorem and l -adic cohomology
- cohomology with compact support
- Poincaré duality
- the Lefschetz trace formula for constructible \mathbb{Q}_l sheaves

Ref: These materials can be found in Milne's lecture [Mi], chapters 18, 21, 23, 24. Another source is Freitag-Kiehl's book [FK88, Chapter I,II], see also SGA 4.5.
Speaker: Võ Quốc Bảo (Institute of Math, Hà Nội)

Talk 3: Picard-Lefschetz theory after Deligne-Katz. This talk and the next talk aim to introduce the basic notions of the theory and to understand the results presented in Sections 4,5 of [Del74]. The topics include

- Lefschetz's pencils, vanishing cycles,
- The monodromy formalism, the Picard-Lefschetz formula

Ref: [SGA7, Chapters XIV, XV] and Freitag-Kiehl's book [FK88, Chapter III, Sections 1-4].
Speaker: Trần Phan Quốc Bảo (Univ. Of Bern)

Talk 4: The global Picard-Lefschetz formula for étale cohomology. This talk is the continuation of the previous talk, and includes the following topics:

- The monodromy mapping and base change
- The global monodromy theory for Lefschetz pencils.

Ref: [SGA7, Chapters XVII, XVIII] and Freitag-Kiehl's book [FK88, Chapter III, Sections 5-7].
Speaker: Nguyễn Thế Hoàng (Thang Long Univ., Hà Nội)

Talk 5: Introduction to Weil conjecture:

- Cohomological interpretation of L-functions, state Weil conjecture (go quickly since we discussed this part already).
- Prove the fundamental inequality
- Poincaré duality in l-adic cohomology and functional equation of L-function.

Ref: [Del74, Sections 2,3] and note of Hindry [Hin10].

Speaker: Đặng Quốc Huy (National Univ. Of Taiwan)

Talk 6: Proof of the Weil conjecture [Del74, Section 6 and 7] and [FR88, Chapter IV].

Speaker: Phùng Hồ Hải (Institute of Math, Hà Nội)

Talk 7: Introduction to the weight theory of l-adic sheaves (Section 1 in [Del80]):

- l-adic sheaves
- weight, pure, mixed weight sheaves.
- filtration of local monodromy (section 1.7-1.8)

Speaker: Nguyễn Đăng Khải Hoàn (Univ. Of Padova)

Talk 8: Proof of the main theorem (Theorem 1 in [Del80]):

- Calculation weight of vanishing cycles.
- Dimension 1 case: Theorem 3.2.3
- General case: Theorem 3.3.1
- Application: structure of mixed sheaves (Section 3.4), and equidistribution theorem (Section 3.5)

Speaker: Đào Văn Thịnh (Institute of Math, Hà Nội)

References:

[Del74] P. Deligne, La conjecture de Weil : I ; Publications mathématiques de l'I.H.É.S., tome 43 (1974), p. 273-307 (translation: <https://www.jmilne.org/math/Documents/DeligneWeill.pdf>)

[Del80] P. Deligne, La conjecture de Weil : II; Publications mathématiques de l'I.H.É.S., tome 52 (1980), p. 137-252.

[FK88] E. Freitag and R. Kiehl, Étale cohomology and the Weil conjecture, Springer 1988.

[Hin10] M. Hindry, introduction to zeta and l-functions from arithmetic geometry and some applications.

[Il21] L. Illusie, Grothendieck and vanishingcycles Tome XXX, no 1 (2021), p. 83 – 115.

[Mi] J. Milne, Lectures on étale cohomology.

[Mor13] B. Morin, Zeta functions of regular arithmetic schemes at $s = 0$.

[SGA4.5] P. Deligne, Cohomologie étale, Lecture Notes in Mathematics, 569, Springer-Verlag, 1977 (translation: <https://www.jmilne.org/math/Documents/DeligneArcata.pdf>)

[SGA7] P. Deligne, A. Grothendieck & N. Katz, Groupes de monodromie en géométrie algébrique, Lecture Notes in Mathematics, vol. 288, 340, Springer, 1972, 1973, Séminaire de Géométrie Algébrique du Bois-Marie 1967–1969 (SGA 7).