

JSPS-VAST Bilateral Joint Research Project Workshop

Singularities, arrangements, and low-dimensional topology, III

Date: 19th (Mon.), 20th (Tue.), December 2022

Place: Institute of Mathematics, Vietnam Academy of Science and Technology

Room: 301, A5

Address: 18 Hoang Quoc Viet Road, Cau Giay District, Hanoi

URL: <https://sites.google.com/keio.jp/jv2021-2024/jv2022-workshop>

Program

19th Dec. (Mon)

Afternoon (Room: 301)

13:50 - 14:40: Kenta Hayano (Keio University)

Lefschetz pencils and multi-objective optimizations

14:50 - 15:40: Tran Nhat Tan (Bochum University)

Flag-accurate arrangements

15:40 - 16:10: *Break*

16:10 - 17:00: Dinh Si Tiep (Institute of Mathematics, VAST)

Lipschitz continuity of tangent directions at infinity

20th Dec. (Tue)

Morning (Room: 301)

10:00 - 10:50: Masaharu Ishikawa (Keio University)

Positive flow-spines and contact 3-manifolds

11:00 - 11:50: Nguyen Thanh Hoang (University of Science and Education, University of Danang)

Geometry of Croke-Kleiner admissible groups

12:00 - 13:50: *Lunch*

Afternoon (Room: 301)

13:50 - 14:40: Mutsuo Oka (Tokyo Institute of Technology)

Zariski pairs of links

14:50 - 15:40: Pho Duc Tai (Hanoi University of Sciences)

Singular curves of genus one in blockchain

15:40 - 16:10: *Break*

16:10 - 17:00: Doan Nhat Minh (Institute of Mathematics, VAST)

Enumerating prime orthogeodesics on decorated punctured surfaces and applications

Organizer:

Tat Thang Nguyen (Institute of Mathematics, VAST)

Masaharu Ishikawa (Keio University)

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Abstract

Kenta Hayano (Keio University)

Title: *Lefschetz pencils and multi-objective optimizations*

Abstract: The purpose of this talk is to (briefly) explain our research projects at present. The two topics in the title are totally independent: a Lefschetz pencil is a family of (possibly singular) codimension-2 submanifolds in a symplectic manifold, which is a generalization of a generic linear 1-system of a projective variety, and multi-objective optimization is a problem minimizing several functions simultaneously, which frequently appears in the real-world problems (e.g. in engineering). In this talk, we will explain what results we have obtained in either (or both if time permits) of the projects.

Tran Nhat Tan (Bochum University)

Title: *Flag-accurate arrangements*

Abstract: We introduce and study the notion of flag-accurate arrangements which form a subclass of both accurate arrangements due to Muecksch-Roehrle (2021) and divisionally free arrangements due to Abe (2016).

Let \mathcal{A} be a free arrangement of rank n . Suppose that for every $d = 1, \dots, n$, the first d exponents of \mathcal{A} – when listed in increasing order – are realized as the exponents of a free restriction of \mathcal{A} to some intersection of reflecting hyperplanes of \mathcal{A} of dimension d . A free arrangement with this natural property involving their free restrictions is called accurate. If the various flats involved in this definition can be arranged in a flag, we call \mathcal{A} flag-accurate.

We present several flag-accurate arrangements arising from reflection groups, root systems, graphs and directed graphs from both theoretical and computational perspectives. In addition to the main result of Muecksch-Roehrle, we show that there are many flag-accurate (hence accurate) arrangements that are not MAT-free. We also give examples of accurate arrangements that are not flag-accurate.

This is joint work with P. Muecksch and G. Roehrle.

Dinh Si Tiep (Institute of Mathematics, VAST)

Title: *Lipschitz continuity of tangent directions at infinity*

Abstract: We relate the set of asymptotic critical values of a polynomial $f: \mathbb{R}^n \rightarrow \mathbb{R}$ with the set of discontinuity of two functions: the first one is a set-valued function which associates to each $t \in \mathbb{R}$ the set of tangent directions at infinity of the fiber $f^{-1}(t)$ and the second one is the $(n - 2)$ -dimensional volume of the first one. (joint work with Pham Tien Son)

Masaharu Ishikawa (Keio University)

Title: *Positive flow-spines and contact 3-manifolds*

Abstract: A contact structure on a closed, oriented, smooth 3-manifold can be studied via its open book decomposition up to contactomorphism, which is the so-called Giroux correspondence. An open book is defined by a pair of a fiber surface with boundary and the monodromy diffeomorphism. Now we like to study contact 3-manifolds by using branched polyhedrons instead of fiber surfaces. A flow-spine is a branched polyhedron with a transversal non-singular flow. We say that a contact structure on a 3-manifold is supported by a flow-spine if it has a contact form whose Reeb flow is a flow of the flow-spine. It can be shown that there exists a correspondence between "positive" flow-spines and contact 3-manifolds up to contactomorphism. In this talk, we first recall the Giroux correspondence and then introduce our results concerning positive flow-spines and contact 3-manifolds. This is a joint work with Ippei Ishii in Keio University, Yuya Koda in Hiroshima University and Hironobu Naoe in Chuo University.

Nguyen Thanh Hoang (University of Science and Education, University of Danang)

Title: *Geometry of Croke-Kleiner admissible groups*

Abstract: Admissible groups were first introduced by Croke-Kleiner to abstract the properties of the graphs of groups structure of the fundamental groups of non-geometric graph manifolds. This talk will focus on the geometry and algebraic properties of Croke-Kleiner admissible groups, generalizing various results from graph manifolds to the setting of Croke-Kleiner admissible groups. We show that abelian subgroups of a Croke-Kleiner admissible group G are separable (and thus G is residually finite), and also give a characterization of strongly quasiconvex subgroups of G . We show that the group G has property (QT), i.e, it acts properly on a finite product of quasi-trees. As applications, we show that almost finitely generated 3-manifold groups have property (QT). Several of these results are joint work with Wenyuan Yang.

Mutsuo Oka (Tokyo Institute of Technology)

Title: *Zariski pairs of links*

Abstract:

Assume that we have a pair of hypersurfaces $V(f) = \{f(\mathbf{z}) = 0\}$ and $V(g) = \{g(\mathbf{z}) = 0\}$ with isolated singularity at the origin. We say $\{V(f), V(g)\}$ is a μ -Zariski pair of hypersurface (respectively μ^* -Zariski pair of hypersurfaces) if they have a same Milnor number μ (respectively a same μ^* -invariant) and a same zeta function of the Milnor fibrations but they belong to different connected components of μ -constant strata (resp. of μ^* -constant strata). They are defined as semi-algebraic sets.

Theorem: Assume that $\{C, C'\}$ is a Zariski pair of irreducible curves with simple singularities. Two links K_f and K_g are diffeomorphic and they have the same Milnor

number and zeta function.

Conjecture: (1) f, g belongs to different connected components of μ -constant strata.

(2) More strongly, they are not diffeomorphic as pairs $(S^5, K_f), (S^5, K_g)$ Conjecture is true in the following cases!

Theorem [with C.Eyral]: $\{f, f'\}$ be a Zariski pair of sextics with simple singularities where $f = 0$ is of torus type without outer singularity and f' of non-torus type. Then $\{g, g'\}$ is a Zariski pair of links where $g = f + z^7, g' = f' + z^7$.

Pho Duc Tai (Hanoi University of Sciences)

Title: *Singular curves of genus one in blockchain*

Abstract: In this talk, we will give a quick introduction of blockchain, classification of singular curves of genus one, and show how these curves appear in the blockchain technology.

Doan Nhat Minh (Institute of Mathematics, VAST)

Title: *Enumerating prime orthogeodesics on decorated punctured surfaces and applications*

Abstract: Recently, Basmajian, Parlier, and Tan established a family of identities on hyperbolic surfaces which generalize some known and important identities like the Basmajian and McShane identities. One fascinating aspect was that by putting grades on the boundary, they were able to obtain non-trivial identities on the thrice-punctured sphere, unlike the case of the previous identities. However, it was in general difficult to describe or enumerate the index sets of these identities. In this talk, I will explain how to use the identities for the thrice-punctured sphere as a tool to enumerate the elements of the index set, essentially reversing the roles of the two. This is possible because of the rigidity of the hyperbolic thrice-punctured sphere, and the deep connection with the modular group, in particular, as manifested geometrically in the Farey tessellation. We believe that this will shed important insights into the geometry of these surfaces, in particular, on the growth rate of the prime ortho-geodesics with respect to length, and also to the problem of whether the multiplicity (with respect to length) of these geodesics is bounded or unbounded. This is joint work with Ser Peow Tan, Yichen Tao, and Chow Boon Wei.