VIETNAM-KOREA JOINT MEETING ON ALGEBRA

Halong bay, January 19-21, 2015

(Organized by IMH-VAST and VIASM)

PROGRAM

Monday, January 19

Morning

10:45 – 11:30 J. Keum (KIAS)
Rational homology projective planes

Afternoon

14:00 – 14:45 S. Kim (Chungwoon University)
Brill-Noether locus of the moduli space of curves

15:00 – 15:45 P. H. Hai (IM-VAST)
On the differential Galois group of a stratified bundle

16:00 – 16:45 Y. Choi (Yeungnam)
Projective normality of the line bundles on a general k-gonal curve

Tuesday, January 20

09:00 – 09:45 J. Park (SNU)
On knot surgery 4-manifolds

10:00 – 10:45 S. Kwak (KAIST)
Classifications of projective varieties in terms of graded Betti numbers

11:00 – 11:45 N. V. Trung (IM-VAST)
Depth and regularity of powers of sums of ideals

Wednesday, January 21

09:00 – 11:00 Discussion

ABSTRACTS

Rational homology projective planes

J. Keum

KIAS

A normal complex projective surface with quotient singularities only is called a Q-homology projective plane if it has the same betti numbers as the complex projective plane. Classification of Q-homology projective planes is an interesting problem in algebraic geometry. It is also related to the Montgomery-Yang Problem that is a topological problem as to circle bundle structures (Seifert bundle structures) on the 5-dimensional sphere. In this talk, I will report recent progress on the problem.

Brill-Noether locus of the moduli space of curves

Seonja Kim

Chungwoon University

Let M(r,d) be the sublocus of $\mathcal{M}_g$ whose general point corresponds to a smooth curve possessing a linear series of dimension r and degree d. If the Brill-Noether number $\rho := g - (r+1)(g-d+r)$ is negative and no less than -3, then every irreducible component of M(r,d) is codimensiom $-\rho$ in $\mathcal{M}_g$. Furthermore if $\rho = -1$ then M(r,d) is an irreducible divisor which is called an Brill-Noether divisor. In this talk, we will investigate the support of M(r,d) with $\rho = -1$ or -2.

On the differential Galois group of a stratified bundle

Phung Ho Hai

Institute of Mathematics, Hanoi

Let X/k be a smooth scheme. A module over the sheaf of differential operators on X, which is coherent as an O_X-module, is called a stratified bundle. Stratified bundles can be seen as generalization of systems of linear differential equations on X. The differential Galois group of this bundle is essentially the group of symmetries of the system of differential equations. In this work we study the relative case: stratified bundles over a smooth scheme over Dedekind ring. In this situation there are two different Galois groups. The difference of these two groups also reflects the nature of the stratified bundle itself. The aim of this talk is to give an introduction in to this theory, providing some examples and announcing some recent results.

Projective normality of the line bundles on a general k-gonal curve

Youngook Choi

Yeungnam University

This is a joint work with Prof. Seonja Kim at Chungwoon University. A very ample line bundle $\mathcal{L}$ on a smooth curve X is said to be normally generated if $H^0(\mathbb{P}^N, \mathcal{O}(m)) \to H^0(X, \mathcal{L}^m)$ is surjective for all $m \geq 0$, where $\mathbb{P}^N := \mathbb{P}H^0(\mathcal{L})^*$. It is well-known that any line bundle on X of degree at least $2g+1$ is normally generated. If X is a trigonal curve of genus $g > 4$ with $2g - m_X \leq \deg\mathcal{L} \leq 2g$, then a nonspecial very ample line bundle $\mathcal{L}$ fails to be normally generated under the following specific condition:

$$\mathcal{L} \simeq \mathcal{K}_X - \beta g^1_3 + R \text{ for some } R \geq 0,$$

where $\beta = 2g - \deg\mathcal{L}$ and m_X is the Maroni invariant of X. In this talk, we discuss conditions for nonspecial line bundles on a general k-gonal curve failing to be normally generated.

On knot surgery 4-manifolds

Jongil Park

Seoul National University

Since the inception of gauge theory, in particular Seiberg-Witten theory, topologists and geometers working on 4-manifolds have developed various techniques and they have obtained many fruitful and remarkable results on 4-manifolds in last 30 years. Among them, a knot-surgery technique introduced by R. Fintushel and R. Stern turned out to be one of most effective techniques to modify smooth structures without changing the topological type of a given 4-manifold. Nevertheless, there are still fundamental problems on knot surgery 4-manifolds to be settled down. For example, it is an intriguing question to know whether a knot surgery 4-manifold determines a prime knot up to mirror, called Fintushel-Stern conjecture on knot surgery 4-manifold, and how many inequivalent Lefschetz fibration structures on it. In this talk first I'd like to review a knot-surgery technique in some details. And then I'll investigate some open problems such as Fintushel-Stern conjecture and Lefschetz fibration structures.

Classifications of projective varieties in terms of graded Betti numbers

Kwak

KAIST

For a projective variety embedded in a projective space, the graded Betti numbers and distribution of them give us an important information of projective varieties. In this talk, we like to introduce basic facts and related questions on the graded Betti numbers to non-experts with modest background. Classification problems on extremal cases with respect to degree and upper bound of Betti numbers will be explained.

Depth and regularity of powers of sums of ideals

Ngo Viet Trung

Institute of Mathematics, Hanoi

This is a joint work with H.T. Ha and T.N. Trung. Given arbitrary homogeneous ideals I and J in polynomial rings A and B over a field k, respectively, we investigate the depth and the Castelnuovo-Mumford regularity of powers of the sum $I + J \subset A \otimes_k B$ in terms of those of I and J. Our results can be used to study the behavior of the depth and regularity functions of powers of an ideal. For instance, we show that such a depth function can take as its values any infinite non-increasing sequence of non-negative integers.