Workshop on
Random Structures and Related Topics

Hanoi, July 06-07, 2022

PROGRAM

Program of Workshop on

Random Structures and Related Topics

[all times are Hanoi time GMT+7]
[venue: A6 building, IMH-VAST]
[zoom codes of sessions will be sent to online participants by email]

Wednesday 06 July

Morning session

09:30 – 10:20 **Van Vu** (Yale University and VinBigdata)
The mathematics behind recommendation systems: "Should I recommend this product for this person ?"

10:20 – 10:30 **Coffee break**

10:30 – 11:20 **Subhro Ghosh** (National University of Singapore)

The unreasonable effectiveness of determinantal processes

11:20 – 11:40 **Son Tran** (Institute of Mathematics Hanoi & National University of Singapore)

Quantitative Marcinkiewicz's theorem and central limit theorems: Applications to spin systems and point processes

11:40 – 12:00 **Quyet Nguyen** (Institute of Mathematics Hanoi, VAST)

12:00 – 13:20 Lunch

Afternoon session

13:20 – 14:10 **Oanh Nguyen** (Brown University)
The number of limit cycles bifurcating from a randomly perturbed center

14:10 – 14:20 **Coffee break**

14:20 – 15:10 **Ke Wang** (Hong Kong University of Science and Technology)

Principle components of spiked covariance matrices

15:10 – 16:00 **Linh Tran** (Fulbright University Vietnam)
Recent progress on the spectral gap of random regular graph

Thursday 07 July

Morning session

09:00 – 09:50 **Hoi Nguyen** (Ohio State University)
Universality for the minimum modulus of random trigonometric polynomials

09:50 – 10:05 **Coffee break**

10:05 – 10:55 **Ji Oon Lee** (Korea Advanced Institute of Science and Technology)
Weak detection in the spiked Wigner models

11:00 – 11:25 **Nhan Nguyen** (University of Virginia)
Variance of real roots of random polynomials with coefficients of polynomial growth

11:25 – 11:50 **Thang Do** (Institute of Mathematics Hanoi, VAST)
On the existence of negative moments for some non-colliding particle systems and their applications

11:50 – 13:30 Lunch

Afternoon session

13:30 – 14:20 **Viet-Hung Pham** (Institute of Mathematics Hanoi, VAST)
Conjunction probability of smooth Gaussian fields

14:20 – 14:35 **Coffee break**

14:35 – 15:25 **Anirban Basak (**Tata institute of fundamental research, India)
Large deviations in sparse random graphs

15:25 – 16:15 **Van Hao Can** (Institute of Mathematics Hanoi, VAST)
Asymptotic behavior of first passage time in frog models

ABSTRACTS

The mathematics behind recommendation systems: “Should I recommend this product for this person ?”

Van Vu
Yale University and VinBigdata

In this talk, we discuss the mathematical core of this problem, starting with the works of E. Candes and T. Tao about 15 years ago, and concluding with new methods and results motivating by modern random matrix theory. The talk will be accessible to undergraduate students with background in science.

The unreasonable effectiveness of determinantal processes

Subhro Ghosh
National University of Singapore)

In 1960, Wigner published an article famously titled "The Unreasonable Effectiveness of Mathematics in the Natural Sciences”. In this talk we will, in a small way, follow the spirit of Wigner’s coinage, and explore the unreasonable effectiveness of determinantal processes (a.k.a. DPPs) far beyond their context of origin. DPPs originated in quantum and statistical physics, but have emerged in recent years to be a powerful toolbox for many fundamental learning problems. In this talk, we aim to explore the breadth and depth of these applications. On one hand, we will explore a class of Gaussian DPPs and the novel stochastic geometry of their parameter modulation, and their applications to the study of directionality in data and dimension reduction. At the other end, we will consider the fundamental paradigm of stochastic gradient descent, where we leverage connections with orthogonal polynomials to design a minibatch sampling technique based on data-sensitive DPPs; with provable guarantees for a faster convergence exponent compared to traditional sampling. Based on the following works.

[1] Gaussian determinantal processes: A new model for directionality in data, with P. Rigollet, Proceedings of the National Academy of Sciences, vol. 117, no. 24 (2020), pp. 13207--13213 (PNAS Direct Submission)
[2] Determinantal point processes based on orthogonal polynomials for sampling minibatches in SGD, with R. Bardenet and M. Lin Advances in Neural Information Processing Systems 34 (Spotlight at NeurIPS 2021)

Quantitative Marcinkiewicz's theorem and central limit theorems: Applications to spin systems and point processes.

Son Tran
Institute of Mathematics Hanoi and National University of Singapore

Abstract: In this talk, we want to introduce our recent work which could be seen as a quantitative version of the classical Marcinkiewicz's theorem. In particular, we obtain quantitative decay estimates on the Kolmogorov-Smirnov distance between a real random variable X and a Gaussian under the condition that the characteristic function does not vanish only on a bounded disk. Our work complements classical works of Ostrovskii, Linnik, Zimogljad and others, as well as recent advances by Michelen and Sahasrabudhe, Eremenko and Fryntov. For applications, our result leads to quantitative central limit theorems applicable to very general and possibly strongly dependent random systems such as discrete spin systems that is based on the theory of Lee-Yang zeros and α-determinantal processes (α ∈ℝ).

On the universality of the superconcentration in mixed p-spin models

Quyet Nguyen
Institute of Mathematics Hanoi, VAST

Superconcentration is the phenomenon that the usual techniques via concentration measures give sub-optimal bounds on the fluctuation of random objects. Consider the mixed p-spin model with general environments such that the covariance of Hamiltonian process is non-negative. In this talk, we aim to assert the universality of the supperconcentration phenomenon. Precisely, we show that the variance of the free energy grows sublinearly in the size of its expectation when the disordered random variable has the first four moments matching to those of the standard normal distribution.

This is joint work with V. H. Can and H. S. Vu.

The number of limit cycles bifurcating from a randomly perturbed center

Oanh Nguyen
Brown University

We consider the average number of limit cycles that bifurcate from a randomly perturbed linear center where the perturbation consists of random (bivariate) polynomials with i.i.d. coefficients. We reduce this problem to the number of real roots of the random polynomial

$$f(x) = \sum_{k=0}^{n} k^{\rho} \xi_k x^k$$

where the ξ_k are independent with mean 0 and variance 1 and $\rho \leq -1/2$ is a constant. In earlier work, Do, Vu, and myself established this number for $\rho > -1/2$ via the universality method which naturally breaks down for $\rho \leq -1/2$. In this talk, we discuss the solution for the $\rho \leq -1/2$.

Joint work with Manjunath Krishnapu and Erik Lundberg.

Principle components of spiked covariance matrices

Ke Wang
Hong Kong University of Science and Technology

Computing the eigenvalues and eigenvectors of a large matrix is a basic task in high dimensional data analysis with many applications in computer science and statistics. In practice, however, data is often perturbed by noise. In this talk, we will focus on the spiked covariance matrix model, a popular and sophisticated model proposed by Johnstone. We will present some recent results on the limiting behaviour of the extreme eigenvalues and eigenvectors of the spiked covariance matrices in the supercritical case.

This talk is based on joint work with Zhigang Bao, Xiucai Ding, and Jingming Wang.

Recent progress on the spectral gap of random regular graph

Linh Tran
Fulbright University Vietnam

Estimating the spectral gap of random regular graph is a long standing problem, started with the works of Alon (1986) and Friedman (2003) for the case that the degree d is fixed. A conjecture by Vu predicts that the same bound would hold for the case that d growing as function of n. Many results have been proved to establish this conjecture in different regimes of d. This talk would survey some of the recent results and their novel approaches to this problem.

Universality for the minimum modulus of random trigonometric polynomials

Hoi Nguyen
Ohio State University

We consider the restriction to the unit circle of random degree-n polynomials with iid coefficients (Kac polynomials). Recent work of Yakir and Zeitouni shows that for Gaussian coefficients, the minimum modulus (suitably rescaled) follows a limiting exponential distribution. We show this is a universal phenomenon, extending their result to arbitrary sub-Gaussian coefficients, such as Rademacher signs. Our approach relates the joint distribution of small values at several angles to that of a random walk in high-dimensional phase space, for which we obtain strong central limit theorems. The case of discrete coefficients is particularly challenging as the distribution is then sensitive to arithmetic structure among the angles.

Based on joint work with Nicholas Cook (Duke University).

Weak detection in the spiked Wigner models

Ji Oon Lee
Korea Advanced Institute of Science and Technology

The spiked Wigner matrix is one of the simplest models for the signal-plus-noise data, where the signal is a rank-1 matrix and the noise is a symmetric random matrix. If the signal-to-noise (SNR) is above a certain threshold, the signal can be reliably detected by the principal component analysis (PCA) or its variants. On the other hand, if the SNR is below the threshold, it is only possible to consider a weak detection which is a hypothesis test between the null model and the alternative. In this talk, I will explain the fundamental limit and efficient algorithms for the weak detection in the spiked Wigner models, which are based on study of random matrices and spin glass models.

This is a joint work with Hye Won Chung (KAIST).

Variance of real roots of random polynomials with coefficients of polynomial growth

Nhan Nguyen
University of Virginia

If we consider a polynomial with random coefficients, then the number of its real roots becomes a random variable and a key problem in the theory of random polynomial is to understand the behavior and the limiting law of this random variable when the degree of the polynomial approaches infinity. To this end, we need to investigate some of its statistical invariants such as its expectation and variance. Despite the large number of prior studies, only a few are about the variance of the number of real roots. In this talk, we compute the large degree asymptotic for the variances of the number of real roots of random polynomials with arbitrary coefficients of polynomial growth.

This is a joint work with Yen Do (University of Virginia, USA).

On the existence of negative moments for some non-colliding particle systems and their applications

Thang Do
Institute of Mathematics Hanoi, VAST

We consider a class of d-dimensional stochastic differential equations that model a non-colliding random particle system. We provide a sufficient condition, which does not depend on the dimension d, for the existence of negative moments of the gap between two particles, and then apply this result to study the strong rate of convergence of the semi-implicit Euler-Maruyama approximation scheme. Our finding improves a recent result of Ngo and Taguchi (Annals of Applied Probability, 2020).

Conjunction probability of smooth Gaussian fields

Viet-Hung Pham
Institute of Mathematics Hanoi, VAST

In this talk, we firstly recall some facts about the distribution of the maximum of Gaussian fields such as: Euler characteristic method, Rice method and the asymptotic formula for the case of non-convex index domain. Then we will present a relevant problem, so-called conjunction probability, founded by Worsley and Friston. We provide the asymptotic formula for the conjunction probability and compare it with the heuristic approximation given by the Euler characteristic method. We also discuss a relevant problem in Integral geometry.

Large deviations in sparse random graphs

Anirban Basak
Tata institute of fundamental research, India

The classical theory of large deviations deals with precise estimates on the probabilities of rare events (e.g. normalized sum of i.i.d. random variables exceed its expectation by a constant factor greater than one) involving linear functions of independent random variables. Beyond the linear setting, possibly the simplest non-trivial interesting question is the large deviations of the triangle counts in Erdos-Renyi graphs. In the past fifteen years there has been a lot of work in this direction. In this talk we will discuss some of them, focusing mainly on the large deviations of (i) regular subgraph counts and (ii) the spectral radius for sparse Erdos-Renyi graphs.

Asymptotic behavior of first passage time in frog models

Van Hao Can
Institute of Mathematics Hanoi, VAST

Frog models are simple but well-known models in the study of the spread of infection. In these models, individuals (also called frogs) move on the integer lattice $\mathbb{Z}^d$ and have one of two states infected (active) and healthy (passive). We assume that at the beginning, there is only one infected frog at the origin, and there are healthy frogs at other sites of $\mathbb{Z}^d$. When a healthy frog encounters an infected one, it becomes infected forever. While the healthy frogs do not move, the infected frogs perform independent simple random walks. In this talk, we will discuss the asymptotic behavior, in particular the law of large numbers, fluctuation, large deviation principles, of the time that a given vertex gets infected.

This talk is based on joint works with Shuta Nakajima (Meiji University) and Naoki Kubota (Nihon University).