

INTERNATIONAL WORKSHOP
Variational Analysis and Optimization Theory

December 19-21, 2017

Hanoi, Vietnam

PROGRAM
&
ABSTRACTS

Institute of Mathematics

Vietnam Academy of Science and Technology

Sponsors

- Vietnam Academy of Science and Technology (VAST)
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Organizing Committee

- **Boris S. Mordukhovich** (Wayne State University)
- **Hoang Xuan Phu** (Institute of Mathematics, VAST)
- **Nguyen Dong Yen** (Institute of Mathematics, VAST)

Location

The workshop takes place at the Institute of Mathematics, in the building A5 on the campus of Vietnam Academy of Science and Technology, No. 18 Hoang Quoc Viet Road, Cau Giay District, Hanoi, Vietnam. Although the main entrance of the campus is No. 18 Hoang Quoc Viet Road, the best way to come to the building A5 and A6 is through the gate 18B Hoang Quoc Viet Road, which is opposite to Academy of National Defense.

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PROGRAM

Tuesday, December 19, 2017

Morning

08:00 – 08:30 **Registration**
2nd floor, **Building A5**,

08:30 – 08:50 **Opening Ceremony**
Grand Hall, 3rd floor, **Building A5**

Session 1

Chair: **Prof. Nguyen Dong Yen**

08:50 – 09:30 **Prof. Le Dung Muu**
(TIMAS, Thang Long University and Institute of Mathematics, VAST)
*A Quasinonexpansive Mapping Fixed Point Approach
to Pseudomonotone Equilibrium Problems*

09:30 – 10:10 **Prof. Nguyen Xuan Tan** (Institute of Mathematics, VAST)
*Quasi-intersection Problems and Fixed Point Theorems
of the Sum of Separately l.s.c and u.s.c Mappings*

10:10 – 10:40 **Coffee break**

Session 2

Chair: **Prof. Le Dung Muu**

10:40 – 11:20 **Prof. Truong Xuan Duc Ha** (Institute of Mathematics, VAST)
*Directional Derivative and Slope of Set-valued Maps,
and Applications in Set Optimization*

11:20 – 12:00 **Prof. Nguyen Nang Tam** (Hanoi University of Pedagogy 2)
*On Solution Existence for Quadratic Programs
in Reflexive Banach Spaces*

12:00 – 13:30 **Lunch**

Afternoon

Session 3

Chair: **Prof. Truong Xuan Duc Ha**

- 13:30 – 14:10 **Dr. Nguyen Huu Tron** (Quy Nhon University)
Directional Hölder Metric Regularity of Set-valued Mappings, and Applications
- 14:10 – 14:50 **Dr. Hoang Ngoc Tuan** (Hanoi University of Pedagogy 2)
DC Algorithms in Hilbert Spaces and Their Applications in Solving Indefinite Quadratic Programs
- 14:50 – 15:20 **Coffee break**

Session 4

Chair: **Prof. Nguyen Xuan Tan**

- 15:20 – 16:00 **Prof. Lam Quoc Anh** (Can Tho University)
On the Stability for Equilibrium Problems and Related Problems
- 16:00 – 16:20 **Vo Thanh Tai** (An Giang University)
On Hölder Continuity of Solution Map of the Parametric Equilibrium Control Problem
- 16:20 – 16:40 **Pham Thanh Duoc** (Vo Truong Toan University)
On Hölder Continuity of Solution Map to Parametric Vector Primal and Dual Equilibrium Problems
- 16:40 – 17:00 **Pham Thi Vui** (Naresuan University, Thailand)
Levitin-Polyak Well-posedness Properties of Optimization Problems by Set Approach

Wednesday, December 20, 2017

Morning

Session 5

Chair: **Prof. Huynh Van Ngai**

08:30 – 09:30 **Prof. Boris S. Mordukhovich** (Wayne State University, USA)
Critical Multipliers in Parametric Variational Systems

09:30 – 10:10 **Prof. Phan Quoc Khanh**
(International University, Ho Chi Minh City)
*Versions of the Weierstrass Theorem for Bifunctions
and the Solution Existence in Optimization*

10:10 – 10:40 **Coffee break**

Session 6

Chair: **Prof. Phan Quoc Khanh**

10:40 – 11:20 **Dr. Nguyen Huy Chieu** (Vinh University)
*Characterization of Tilt Stability via Subgradient Graphical Derivative,
with Applications to Nonlinear Programming*

11:20 – 12:00 **Dr. Pham Duy Khanh**
(Ho Chi Minh City University of Education)
*Second-order Characterizations of $C^{1,1}$ -smooth Pseudoconvex
and Quasiconvex Functions*

12:00 – 13:30 **Lunch**

Afternoon

Session 7

Chair: **Prof. Boris S. Mordukhovich**

- 13:30 – 14:10 **Prof. Huynh Van Ngai** (Quy Nhon University)
*Inverse Function Theorems for Multifunctions
in Graded Fréchet Spaces*
- 14:10 – 14:50 **Dr. Tran Thai An Nghia** (Oakland University, USA)
Q-Linear Convergence of Iterative Soft-thresholding Algorithm
- 14:50 – 15:10 **Nguyen Nguyen Truc Dao** (Wayne State University, USA)
Optimal Control of a Perturbed Sweeping Process
- 15:10 – 15:40 **Coffee break**

Session 8

Chair: **Dr. Nguyen Huy Chieu**

- 15:40 – 16:20 **Dr. Nguyen Van Luong** (Hong Duc University)
*Variational Analysis for Minimal Time Function
with Nonconvex Dynamics*
- 16:20 – 16:40 **Vu Thi Huong** (Institute of Mathematics, VAST)
Differentiability Properties of a Parametric Consumer Problem
- 16:40 – 17:00 **Tran Van Nam** (University of Danang, campus in Kon Tum)
Optimization with flexible objective
- 17:00 – 17:20 **Do Ngoc Hong** (Wayne State University, USA)
*Critical Multipliers via Second-order Generalized Differentiation
of a Subclass of Piecewise Linear-quadratic Functions*
- 18.00 – 20.00 **Conference Dinner**

Thursday, December 21, 2017

Morning

Session 9

Chair: **Dr. Tran Thai An Nghia**

- 08:30 – 09:10 **Dr. Cao Hoang Tan** (SUNY Korea, Korea)
*Optimal Control of a Nonconvex Perturbed Sweeping Process
with Applications to the Crowd Motion Model*
- 09:10 – 09:50 **Prof. Nguyen Dong Yen** (Institute of Mathematics, VAST)
*Second-order Subdifferentials and Optimality Conditions
for C^1 -smooth Optimization Problems*
- 09:50 – 10:20 **Coffee break**
- 10:30 – 18:00 **Excursions**

ABSTRACTS

On the Stability for Equilibrium Problems and Related Problems

Lam Quoc Anh¹

Abstract: In this report, we consider the equilibrium problem and its extensions. We study several qualitative properties of solution mappings to such problems, including semicontinuity in the sense of Berge and Hausdorff and the Hölder/Lipschitz continuity of solution mappings. The study of well-posedness for these problems is also investigated. Some efficient approaches to studying the stability conditions and well-posedness such as gap functions, scalarization methods are also discussed.

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On Hölder Continuity of Solution Maps to Parametric Vector Primal and Dual Equilibrium Problems

Lam Quoc Anh¹, Pham Thanh Duoc², and Tran Ngoc Tam^{3,4}

Abstract: In this talk, we first propose some kinds of the strong convexity (concavity) for vector functions. Then we apply these assumptions to establish the sufficient conditions for the Hölder continuity of solution maps of the vector primal and dual equilibrium problems in metric linear spaces. As applications, we derive this Hölder continuity of solution maps of vector optimization problems and vector variational inequality problems. Our results are new and include the recent existing ones in the literature.

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Levitin-Polyak Well-posedness Properties of Optimization Problems by Set Approach

Lam Quoc Anh¹, Tran Quoc Duy², Rabian Wangkeeree³, and Pham Thi Vui⁴

Abstract: In this talk, we consider optimization problems which the solution criterion based on set approach. The Levitin-Polyak (LP) well-posedness for set optimization problems involving several types of set order relations is investigated. Both pointwise and global LP well-posedness are considered. We establish criteria and characterizations of these kinds of well-posedness for set optimization problems. The relationship between different types of LP well-posedness is studied. Moreover, the link between stability of approximating problem and well-posedness of the set optimization problem is given.

References

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On Hölder Continuity of Solution Map of the Parametric Equilibrium Control Problem

Lam Quoc Anh¹, Tran Ngoc Tam², and Vo Thanh Tai³

Abstract: In this report, we study the parametric equilibrium control problem with linear state equation and control constraints. Sufficient conditions for Hölder continuity of solution maps are established. In addition, we also provide some examples to illustrate advantages of the results over existing ones.

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Characterization of Tilt Stability via Subgradient Graphical Derivative with Applications to Nonlinear Programming

Nguyen Huy Chieu¹

Abstract: This talk will present some of our recent results on tilt stability in finite dimensional optimization. We show a new characterization of tilt-stable local minimizers for a broad class of unconstrained optimization problems in terms of a uniform positive-definiteness of the subgradient graphical derivative of the objective function around the point in question. Applying this result to nonlinear programming under the metric subregularity constraint qualification yields new second-order characterizations and several new sufficient conditions for tilt stability. In particular, each stationary point of a nonlinear programming problem satisfying the metric subregularity constraint qualification is a tilt-stable local minimizer if the classical strong second-order sufficient condition holds.

This talk is based on joint work with L. V. Hien (Ha Tinh University) and Tran T. A. Nghia (Oakland University).

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Second-order Subdifferentials and Optimality Conditions for C^1 -smooth Optimization Problems

Nguyen Huy Chieu¹, Gue Myung Lee², and Nguyen Dong Yen³

Abstract: We investigate the possibility of using the Fréchet and limiting second-order subdifferentials to characterize locally optimal solutions of C^1 -smooth unconstrained minimization problems. We prove that, for a C^1 -smooth function of one real variable or a C^1 -smooth function on a Banach space with its derivative being calm at the reference point, the positive semi-definiteness of its Fréchet second-order subdifferential at the point in question is a necessary optimality condition, while it is not true for the limiting counterpart. However, the limiting second-order subdifferential of a $C^{1,1}$ -smooth function on \mathbb{R}^n at a local minimizer is positively semi-definite along some of its selection. We also show that, for a C^1 -smooth function on an Asplund space, the positive semi-definiteness of its Fréchet second-order subdifferential around a stationary point is sufficient for this point to be a local minimizer of the function. Besides, a sufficient condition via the Fréchet second-order subdifferential for a point to be a tilt stable minimizer is given.

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Optimal Control of a Perturbed Sweeping Process

Nguyen Nguyen Truc Dao¹

Abstract: This paper deals with an optimal control problem for a perturbed sweeping (Moreau) process, where control function is in additive perturbations on the right-hand side of the dissipative differential inclusion without changing the moving set and merely measurable without any Lipschitzian. It should be emphasized that the velocity mapping in the differential inclusions under consideration is highly non-Lipschitz, unbounded and the control is just measurable, which cannot be treated by means of known results in optimal control for differential inclusions. To overcome such principal issues, we pursue a twofold aim. Firstly, we develop here the method of discrete approximations married with catching-up algorithm and combine it with appropriate generalized differential tools of modern variational analysis, which allows us to adequately replace the original optimal control problem by a sequence of well-posed finite-dimensional optimization problems whose optimal solutions strongly converge to that of the original controlled perturbed sweeping process. Second, we use this direct method to obtain nondegenerate necessary optimality conditions for the so-called intermediate relaxed local minimum that takes an intermediate place between the classical concepts of strong and weak minima of the controlled sweeping process expressed entirely via the problem data. Furthermore, the established necessary optimality conditions for the sweeping process are illustrated by several examples.

This talk is based on joint work with Giovanni Colombo and Boris Mordukhovich.

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Directional Derivative and Slope of Set-valued Maps and Applications in Set Optimization

Truong Xuan Duc Ha¹

Abstract: A directional derivative and a slope for a set-valued map are defined based on a Hausdorff-type distance with respect to an ordering cone between sets in a Banach space. Some applications of these concepts in set optimization (characterization of convexity and Lipschitzity, conditions for optimality and error bounds) are presented.

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Critical Multipliers via Second-order Generalized Differentiation of a Subclass of Piecewise Linear-quadratic Functions

Do Ngoc Hong¹

Abstract: It has been well recognized that critical multipliers, the notion of which developed by Izmailov and Solodov for classical Karush-Kuhn-Tucker (KKT) systems, are largely responsible for slow convergence of major primal-dual algorithms of optimization. Recently their notion has been extended to a general framework of constrained optimization including composite optimization, minimax problems, etc. This talk concerns the critical multipliers for variational systems of a major subclass of piecewise linear-quadratic functions. Implementing a comprehensive second-order study of this class, we obtain complete characterizations of critical and noncritical multipliers via the problem data. It is shown that noncriticality is equivalent to a certain calmness property of a perturbed variational system. These results can be applied to study the vanishment of critical multipliers under the fulfilment of the full stability of local minimizers in problems of composite optimization.

This talk is based on joint work with Boris Mordukhovich and Ebrahim Sarabi.

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Differentiability Properties of a Parametric Consumer Problem

Vu Thi Huong¹, Jen-Chih Yao², and Nguyen Dong Yen³

Abstract: We study the budget map and the indirect utility function of a parametric consumer problem in a Banach space setting by some advanced tools from set-valued and variational analysis. The Lipschitz-likeness and differentiability properties of the budget map, as well as formulas for finding subdifferentials of the indirect utility function are obtained. Our investigation is mainly based on the paper by Mordukhovich [J. Global Optim. 28 (2004), 347–362] on coderivative analysis of variational systems and the paper of Mordukhovich, Nam, and Yen [Math. Program. 116 (2009), 369–396] on subgradients of marginal functions.

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Second-order Characterizations of $\mathcal{C}^{1,1}$ -smooth Pseudoconvex and Quasiconvex Functions

Pham Duy Khanh¹ and Vo Thanh Phat²

Abstract: Some criteria for the pseudoconvexity and the quasiconvexity of $\mathcal{C}^{1,1}$ -smooth functions are derived in terms of Fréchet and Mordukhovich second-order subdifferentials.

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Versions of the Weierstrass Theorem for Bifunctions and the Solution Existence in Optimization

Phan Quoc Khanh¹

Abstract: We propose an approach to existence studies based on new purely set-theoretic notions of anti-cyclicity and cyclic anti-quasimonotonicity and a given topological setting, without linear or (generalized) convexity structures. We first establish the Weierstrass extreme-value theorem for bifunctions and its equivalent versions (with quite different formulations). Next, we apply these results to obtain sufficient conditions for the solution existence of various optimization-related problems. The proof technique in our unified study of existence is simple and elementary, but seems to be applicable also to many other problems.

This talk is based on a joint work with Nguyen Hong Quan.

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DC Algorithms in Hilbert Spaces and Their Applications in Solving Indefinite Quadratic Programs

Yong Do Lim¹, Hoang Ngoc Tuan², and Nguyen Dong Yen³

Abstract: In this talk, we consider the DC Algorithms in Hilbert spaces and the convergence of two typical types of them, the *Projection DC decomposition algorithm* and the *Proximal DC decomposition algorithm*, are applied for solving an indefinite quadratic programming problem under linear constraints in Hilbert spaces. It is proved that, any iterative sequence generated by either one of the mentioned algorithms R -linearly converges to a Karush–Kuhn–Tucker point of the problem under some mild conditions. The stability of these algorithms is also established.

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Variational Analysis for Minimal Time Function with Nonconvex Dynamics

Nguyen Van Luong¹

Abstract: In this talk, we consider the minimal time function associated with nonconvex dynamics. Several properties concerning continuity, convexity, Lipschitz behavior and subdifferent calculus are presented. We then apply the results to study properties of the minimum time function with the dynamics is a finite union of convex sets.

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Critical Multipliers in Parametric Variational Systems

Boris Mordukhovich¹

Abstract: We introduce the notions of critical and noncritical multipliers for parametric variational systems, which include those arising in problems of nonlinear programming, composite and minimax constrained optimization, conic programming, etc. These notions extend the corresponding ones developed by Izmailov and Solodov for the classical KKT system. It has been well recognized that critical multipliers are largely responsible for slow convergence of major primal-dual algorithms of optimization. Based on advanced tools and results of second-order variational analysis and generalized differentiation, we obtain efficient characterizations of critical and noncritical multipliers in terms of the given data of the problems under consideration. Furthermore, it is shown that critical multipliers can be ruled out by full stability of local minimizers in various classes of composite optimization and conic programming problems. We also establish connections between noncriticality of multipliers and appropriate notions of calmness and isolated calmness for solutions maps to parametric variational systems.

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A Quasinonexpansive Mapping Fixed Point Approach to Pseudomonotone Equilibrium Problems

Le Dung Muu¹

Abstract: Some problems such as optimization, variational inequality, Kakutani fixed point and Nash-equilibrium problems can be formulated in the form of an equilibrium problem defined by the Nikaido-Isoda-Fan inequality. In this talk, we present a fixed point approach of contractive and (generalized) nonexpansive mappings to strongly pseudo and (generalized) monotone equilibrium problems. The approach allows one to apply fixed point methods of contractive and (generalized) nonexpansive mappings to solve some classes of generalized monotone equilibrium problems.

Key words. Convexity, pseudomonotonicity, quasi-nonexpansive mapping, fixed point approach, equilibria.

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Optimization with Flexible Objectives and Constraints

Tran Van Nam¹ and Imme van den Berg²

Abstract: In this work we model uncertainties in optimization problems by *neutrices*-convex subgroups of the nonstandard analysis real line and *external numbers* - the sum of a real number and a neutrix. An external number is a set of nonstandard real numbers relatively close to a given real number, being stable under small perturbations and expressing some flexibilities. The calculus of external numbers may be seen as a model of propagation of errors [1], [2]. A function with values in external numbers is called a *flexible function*.

We shall present necessary and sufficient conditions for the existence of (near) optimal solutions and their characteristics for both linear programming and nonlinear optimization problems with flexible objectives and constraints.

Firstly, we consider linear programming problems with flexible objectives and constraints.

Secondly, we investigate nonlinear optimization problems with flexible objective functions. The Lagrange multiplier method will be also mentioned.

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Inverse Function Theorems for Multifunctions in Graded Fréchet Spaces

Huynh Van Ngai¹

Abstract: The inverse function theorem is one of the central components of the classical and the modern variational analysis and an essential device to solving nonlinear equations. The inverse function theorem or its variants known as the implicit function theorem or the rank theorem have been established originally in Euclidean spaces and then extended to the Banach space setting. Outside this setting, for instance in Fréchet spaces, it is known that the inverse function theorem generally fails. This is the reason why another form of inverse function theorem, nowadays called the Nash–Moser theorem is used as a powerful tool to prove local existence for non-linear partial differential equations in spaces of smooth functions.

Some inverse theorems of Nash–Moser type have also been proved for functions between Fréchet spaces, that are supposed to be tame, an additional property guaranteeing that the semi-norms satisfy some interpolation properties, or that allow the use of smoothing operators as introduced by Nash. To overcome the loss of derivatives, these additional properties in Fréchet spaces allow Newton’s method on which the Nash–Moser type inverse function theorems are based to converge. Recently, Ekeland produced a new result within a class of spaces much larger than the one used in the Nash–Moser literature.

In this talk, we present some inverse function theorems and implicit function theorems for set-valued mappings between Fréchet spaces. The proof relies on Lebesgue’s Dominated Convergence Theorem and on Ekeland’s Variational Principle. An application to the existence of solutions of differential equations in Fréchet spaces with non-smooth data is given.

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Directional Hölder Metric Regularity of Set-valued Mappings and Applications

Huynh Van Ngai¹, Nguyen Huu Tron², and Michel Théra³

Abstract: This paper sheds new light on regularity of multifunctions through various characterizations of directional Hölder/Lipschitz metric regularity which are based on the concepts of strong slope, coderivative. By using these characterizations, we show that directional Hölder/Lipschitz metric regularity is stable when the multifunction under consideration is perturbed suitably. Applications of directional Hölder/Lipschitz metric regularity to investigate the stability and the sensitivity analysis of parameterized optimization problems are also discussed.

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Q -Linear Convergence of Iterative Soft-Thresholding Algorithm

Tran Thai An Nghia¹

Abstract: Iterative soft-thresholding algorithm is a well-known and efficient algorithm for solving nonsmooth convex optimization problems. The most popular complexity of this method is $O(k^{-1})$. However, in practice, numerical experiments usually show evidences of linear convergences. In this talk, I will explain this phenomenon by using tools of generalized differentiations and variational analysis. Local and global linear convergences of this method in some special classes such as Lasso, nuclear-norm, TV-norm regularized optimization problems, feasibility problems, and support vector machine are also discussed. Our approach also reveals a universal technique to solution uniqueness of many structured optimization problems.

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On Solution Existence for Quadratic Programs in Reflexive Banach Spaces

Nguyen Nang Tam¹

Abstract: In this talk we discuss quadratic programs in Banach spaces and propose sufficient conditions for the solution existence of quadratic programs under finitely many convex quadratic constraint in reflexive Banach spaces.

Key words. Quadratic program in Banach spaces, solution existence, Legendre form, recession cone.

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Optimal Control of a Nonconvex Perturbed Sweeping Process with Applications to the Crowd Motion Model

Cao Hoang Tan¹

Abstract: This talk concerns optimal control of discontinuous differential inclusions of the normal cone type governed by a generalized version of the Moreau sweeping process with control functions acting in both nonconvex moving sets and additive perturbations. This is a new class of optimal control problems in comparison with previously considered counterparts where the controlled sweeping sets are described by convex polyhedra. Besides a theoretical interest, a major motivation for our study of such challenging optimal control problems with intrinsic state constraints comes from the application to the crowd motion model in a practically adequate planar setting with nonconvex but prox-regular sweeping sets. Based on a constructive discrete approximation approach and advanced tools of first-order and second-order variational analysis and generalized differentiation, we establish the strong convergence of discrete optimal solutions and derive a complete set of necessary optimality conditions for discrete-time and continuous-time sweeping control systems that are expressed entirely via the problem data. We also discuss how to apply such a set of conditions to solve the controlled problems associated with the crowd motion model of traffic flow in the planar settings.

This talk is based on the joint research with Boris Mordukhovich (Wayne State University, Detroit, MI, USA).

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Quasi-intersection Problems and Fixed Point Theorems of the Sum of Separately l.s.c and u.s.c Mappings

Nguyen Xuan Tan¹

Abstract: In this paper, we apply our new results on quasi-variational inequality problems to quasi-intersection problems. Some sufficient conditions on the existence of solutions to these problems are shown. In particular, we establish several results on the existence of solutions to generalized quasi-equilibrium problems and fixed points of the sum of separately lower and separately upper semi-continuous (separately lower and separately lower semi-continuous, separately upper and separately upper semi-continuous) mappings. These generalize some well-known fixed point theorems previously obtained by F. E. Browder and Ky Fan, X. Wu, L. J. Lin, Z. T. Yu, and other authors.

Let X, Y and Z be Hausdorff locally convex topological vector spaces over reals, $D \subset X, K \subset Z$ be nonempty subsets. Given multi-valued mappings $P : D \times K \rightarrow 2^D, Q : D \times K \rightarrow 2^K$ and $G, H : D \times K \rightarrow 2^{X \times Z}$, we are interested in the problem of finding $(\bar{x}, \bar{y}) \in D \times K$ such that

$$\begin{aligned}\bar{x} &\in P(\bar{x}, \bar{y}); \\ \bar{y} &\in Q(\bar{x}, \bar{y}); \\ G(\bar{x}, \bar{y}) \cap H(\bar{x}, \bar{y}) &\neq \emptyset.\end{aligned}$$

It is clear that quasi-intersection problems are generalizations of variational inequalities and optimization problems, including also optimization-related problems such as fixed point, complementarity problems, Nash equilibrium, minimax problems, etc. In the last decade, a number of generalizations of these problems to different directions such as quasi-equilibrium problems with constraint sets depending on parameters, quasi-variational and quasi-equilibrium inclusion problems with multi-valued data, have been proposed. Quasi-intersection problems also encompass a large family of problems in applied mathematics including quasi-optimization problems, quasi-variational inclusion, quasi-equilibrium problems, quasi-variational relation problems, etc.

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