Title: Geometric Quantization.

Instructor: Daniel Burns (University of Michigan)

Summary: Classical mechanics has a natural framework (Hamiltonian mechanics) in symplectic geometry. Quantum mechanics replaces the phase spaces of classical mechanics with Hilbert spaces and operators on them. This "quantization" procedure is not well-defined: there does not seem to be any "canonical" method for associating the quantum representation to a classical system. Geometric quantization is one mathematical attempt to provide a method for quantizing a classical system canonically, starting from the underlying geometry of classical mechanics.

In recent decades there have been enormous new developments in symplectic geometry, and now it seems there are constraints on quantum mechanics coming from the "new" symplectic geometry of the underlying classical mechanical system. Work of Leonid Polterovich and his collaborators have shown this within the framework of geometric quantization in Hilbert spaces of holomorphic functions. We will try to give a flavor of these developments in the final lecture.

Given the brief time available to us, we will often forego detailed proofs when appropriate examples will convey the sense of the proof. References will be given, or distributed, if openly available, and course notes will be posted ahead of lectures. Open problems will be highlighted along the way.

Here is a brief schedule of the minicourse.

Lecture 1: Symplectic manifolds, classical mechanics.

Lecture 2: Kähler manifolds: examples, line bundles, Chern forms.

Lecture 3: Spaces of holomorphic sections, Bergman projectors.

Lecture 4: Berezin-Toeplitz operators, canonical commutation relations.

Lecture 5: Symplectic rigidity and the "quantum speed limit".

This framework should leave a little time for questions and wrap-up at the end of the course.

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