The Riemann zeta function $\zeta(s)$, first written down by Euler, is of basic importance for the study of the distribution of prime numbers. The Riemann hypotheses, arguably the most famous unsolved conjecture in mathematics makes the simple prediction that the zeroes of $\zeta(s)$ with $0 \le \mathrm{Re}\, s \le 1$ lie on the line $\mathrm{Re}\, s = 1/2$. Knowing this would have great consequences for the study of prime numbers. The special values of $\zeta(s)$ for integers $s = n$ have also attracted a great deal of interest since Euler's proof that $\zeta(2) = \sum_{n=1}^{\infty} n^{-2} = \pi^2/6$. Similar formulas exist for even values $s = 2\nu$ with $\nu \ge 1$ but an understanding of the nature of the values $\zeta(3), \zeta(5), \ldots$ emerged only much later in the work of Borel on higher regulators of algebraic K-groups. The theory of the Riemann zeta function was extended by Dedekind to rings of integers in number fields. It governs the distribution of the prime ideals of the number ring. Around 1920 Artin studied an analogous zeta function for certain smooth projective curves over finite fields and verified an analogue of the Riemann hypotheses for them. More generally, Hasse and Weil introduced a zeta function for all algebraic schemes over $\mathrm{spec}\, \mathbb{Z}$ (in modern terminology). Hasse proved the Riemann hypotheses for elliptic curves over finite fields and Weil succeeded to do the same for all smooth projective curves over finite fields. Later Weil studied the Hasse-Weil zeta function for higher dimensional varieties over finite fields and in the smooth projective case formulated his famous Weil conjectures. He also suggested a path to approach his conjectures by interpreting the zeta function as an alternating product of characteristic polynomials of the Frobenius homomorphism acting on an as yet undefined cohomology theory for algebraic varieties with similar formal properties as singular cohomology of manifolds. A good deal of Grothendieck's revolutionary new approach to algebraic geometry via schemes was devoted to the development of such a "Weil cohomology" for algebraic varieties in order to prove the Weil conjectures. This program was successful and culminated in Deligne's proof of the generalized Riemann hypotheses for smooth projective varieties over finite fields in 1973. He used l-adic cohomology for his proof. Later developments also allowed a proof via crystalline cohomology, which was the second Weil cohomology that Grothendieck invented. While l-adic cohomology is based on the study of étale coverings of algebraic varieties, crystalline cohomology is closer in spirit to de Rham's idea of describing cohomology in terms of differential forms. Over finite fields the cohomological expression of the zeta-function has also been used to prove formulas for their special values. Moreover, cohomology has been a vital tool in Drinfeld's and Lafforgue's proofs of the Langlands conjecture for GL_2 resp. GL_n over function fields of curves. For algebraic schemes over $\operatorname{spec} \mathbb{Z}$ with infinitely many residue characteristics the situation is much less satisfactory. There are precise conjectures on the Hasse-Weil zeta function concerning its analytic continuation, functional equation and the location of its zeroes and poles. Moreover Birch-Swinnerton Dyer, Bloch, Beilinson, Kato, Lichtenbaum, Flach-Morin, Soulé and others have predicted the orders of vanishing and the leading terms of the zeta function using cohomology theories (motivic, Deligne, syntomic, Weil-étale and several others). All progress on these conjectures today relies on first expressing the Hasse-Weil zeta function by a product of automorphic L-functions and using ingenious arguments which are specific for the situation. What is missing, even for the Riemann zeta function is an infinite dimensional complex cohomology theory with an operator that could serve the same purposes for general algebraic schemes over $\operatorname{spec} \mathbb{Z}$ as l-adic cohomology with Frobenius endomorphisms for varieties over finite fields. We will explain the formalism that such an infinite dimensional cohomology theory should satisfy and some of its expected properties. We will also mention recent ideas by ourselves and by Clausen–Scholze on a geometry for algebraic schemes over $\operatorname{spec} \mathbb{Z}$ which carries an $\mathbb{R}^{>0}$-action instead of Frobenius that would give rise to the desired cohomology.