

SEMINAR ON LOCAL SYSTEMS

1 Introduction

This is a preparation seminar for the lecture of Prof. Hélène Esnault this December. We will follow her new book [Esn23] to learn some basic concepts, conjectures, and ideas related to Local systems in Algebraic–Arithmetic geometry. Here is our plan.

2 Plan

Talk 1: 07-12-2023, time: 14-15:15.

Title: Introduction to flat connections, de Rham cohomology, and local systems.

Abstract: This talk will provide some basic concepts: flat connection (stratified bundle); de Rham cohomology, and local system. For de Rham cohomology, we concentrate on the case of characteristic p . If time permits, we will recall the Riemann-Hilbert correspondence. References: [Kat70], [Kat72].

Speaker: Pham Thanh Tam

Talk 2: 07-12-2023, time: 15:30-16:45.

Title: Kronecker’s rationality criteria and Grothendieck p -conjecture.

Abstract: This talk will follow closely sections 2.1-2.2 in [Esn23]. The key observation is Claim 2.9 and Claim 2.11. References: [Esn23] and therein especially [Kat70] and [And04].

Speaker: Nguyen Quang Khai

Talk 3: 07-12-2023, time: 17-18:15.

Title: Introduction to Gauss-Manin connection and Hodge F -filtration.

Abstract: We follow [Kat72, Section 1] to understand Hodge F -filtration of a Gauss-Manin connection. After that will be the main task: prove Claim 2.13 in the book [Esn23] (hint: use 1.4.1.7 in [Kat72]).

Speaker: Vo Quoc Bao

Talk 4: 14-12-2023, time: 14-15:15.

Title: Malcev-Grothendieck theorem and its variants

Abstract: This talk starts with the Malcev-Grothendieck theorem (over \mathbb{C}) which roughly says that the étale fundamental group controls the number of local systems of fixed rank. We also have its characteristic p version: Gieseker’s conjecture, which was proved by Esnault-Mehta. In the case of smooth projective schemes over perfect field of positive characteristic, a variant of Malcev-Grothendieck theorem for the crystalline fundamental groups was conjectured by de Jong. References: [Esn23, 3.3 and 3.4] and references therein, especially [EM10, ES18].

Speaker: Tran Phan Quoc Bao

Talk 5: 14-12-2023, time: 15:30-16:45.

Title: Grothendieck's specialization homomorphism and the lifting problem of schemes in positive characteristic

Abstract: This talk will first explain Grothendieck's specialization homomorphism of the étale fundamental group. References: [SGA1, Exposé XIII, 2.10]. This map is used to compare fundamental group schemes in zero and positive characteristic and whence derived properties of the étale fundamental group in positive characteristic. The results obtained can be used to address the lifting problem: what are the obstructions to lifting a scheme in characteristic p to one in characteristic 0. The main references are Sections 4 and 5 in [Esn23].

Speaker: Nguyen The Hoang

Talk 6 14-12-2023, time: 17-18:15.

Title: Geometric local system and Deligne's companion conjecture.

Abstract: This talk aims to introduce Deligne's companion conjecture and its proof in the case of dimension 1 ([Laf02]). If time permits, we will discuss higher dimensional proof given by Drinfeld [Dri12]. Some key concepts will be recalled: geometric local system, and companion.

Speaker: Dao Van Thinh

References

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