Block seminar on Harmonic Bundles

(Institute of Mathematics and Ha Long, Quang Ninh)

August 21–24, 2025

This block seminar is designed to provide an introduction to the basic concepts of harmonic bundles. It will take place from August 21 to 24, 2025, and will consist of 8 talks. The sessions on August 21 will be held in Room 612-A6 at the Institute of Mathematics (VAST), while the remaining sessions, from August 22 to 24, will take place in Ha Long. The seminar will be conducted in a hybrid format, allowing for both in-person and online participation. Details are as follows:

Talk 1: Introduction to connections in complex vector bundles.

Speaker:	Vo Quoc Bao
Time:	14h00-15h30, August 21, 2025
Venue:	Room 612-A6, Institute of Mathematics, VAST
Reference:	Chapters 1 - 3 in [Kob87]

- Recall definitions and properties of connections (Chap 1 (1.1–1.6) in [Kob]).
- Define Chern classes (Chap 2 (2.1–2.3)).
- Some cohomological comparison and Serre duality (Chap 3 (3.2)).

Talk 2: Introduction to Einstein-Hermitian vector bundles.

Speaker:	Dinh Quy Duong
Time:	15h45-17h15, August 21, 2025
Venue:	Room 612-A6, Institute of Mathematics, VAST
Reference:	Chapters 4 - 5 in [Kob87]

- Explain Einstein condition and study Chern class of Einstein-Hermitian vector bundles (Chap 4 (4.1, 4.4)).
- Relate to stable vector bundles over compact Kähler manifolds.
- Prove general case via extension to coherent sheaves (Chap 5 (5.1–5.8)).

Talk 3: Introduction to harmonic bundles.

Speaker:	Vo Anh Duc
Time:	8h00-9h30, August 23, 2025
Venue:	Ha Long, Quang Ninh
Reference:	Section 1 in [Sim92]

- Correspondence between flat bundles and Higgs bundles.
- Define harmonic bundles (p.18 in [Sim92]) and motivation via Corlette [Cor88].
- Prove the non-abelian Hodge theorem.

Talk 4: Kähler identity and compactness.

Speaker: Nguyen The Hoang

Time: 9h45-11h15, August 23, 2025

Venue: Ha Long, Quang Ninh

Reference: Section 2 in [Sim92]

- First-order and higher-order Kähler identities.
- Prove de Rham = Dolbeault cohomology.
- Prove compactness: bounded harmonic bundles form a compact set.

Talk 5: Good formal property of meromorphic ρ-flat bundles.

Speaker: Tran Phan Quoc Bao

Time: 14h00-15h30, August 23, 2025

Venue: Ha Long, Quang Ninh

Reference: Chapter 2 in [Mo11]

- Define good set of irregular values.
- Study unramified good lattices after formal completion (Prop 2.4.1).
- Introduce good filtered ρ-flat bundles.
- Mention Deligne-Malgrange lattice.
- Define λ-flat bundles.

Talk 6: Stokes structure of unramifiedly good ρ-flat bundle.

Speaker: Phung Ho Hai

Time: 15h45-17h15, August 23, 2025

Venue: Ha Long, Quang Ninh

Reference: Chapter 3 in [Mo11]

- Describe full/partial Stokes filtrations.
- Show recovery of meromorphic flat bundle from Stokes structure.
- Define associated graded flat bundle.

Talk 7: Full Stokes data and Riemann-Hilbert-Birkhoff correspondence.

Speaker: Dao Van Thinh

Time: 8h00-9h30, August 24, 2025

Venue: Ha Long, Quang Ninh

Reference: Chapter 4 in [Mo11]

- Define full Stokes data.

- State correspondence with unramifiedly good lattices.

- Applications to extension and deformation of good lattices.

Talk 8: Asymptotic behavior of wild harmonic bundles.

Speaker: Pham Thanh Tam

Time: 9h45-11h15, August 24, 2025

Venue: Ha Long, Quang Ninh

Reference: Chapter 7 in [Mo11]

- Define wild harmonic bundle (Section 7.1).

- Prove wild version of Simpson's estimates (Sections 7.2–7.3).

- Define and study the prolongment PE_λ (Theorem 7.4.3, 7.4.5).

References

[Mo11] MOCHIZUKI, T., *Wild harmonic bundles and wild pure twistor D-modules*, Astérisque, no. 340 (2011).

[Cor88] CORLETTE, K., Flat G-bundles with canonical metrics, *Diff. Geom.*, 28 (1988), 361–382.

[Kob87] KOBAYASHI, S., *Differential Geometry of Complex Vector Bundles*, Princeton University Press, (1987).

[Sim92] SIMPSON, C., Higgs bundles and local systems, *Inst. Hautes Études Sci. Publ. Math.* No. 75 (1992), 5–95.