

Control Simulation Experiments with the Lorenz-96 Model

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Outline

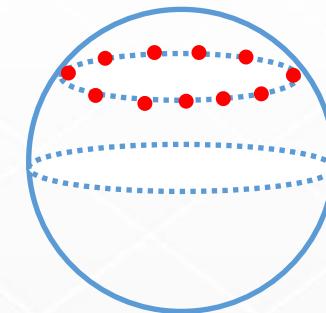
- Lorenz 96 model
- Extreme events
- Experiments
- Full control
- Partial control

Lorenz 96 Model

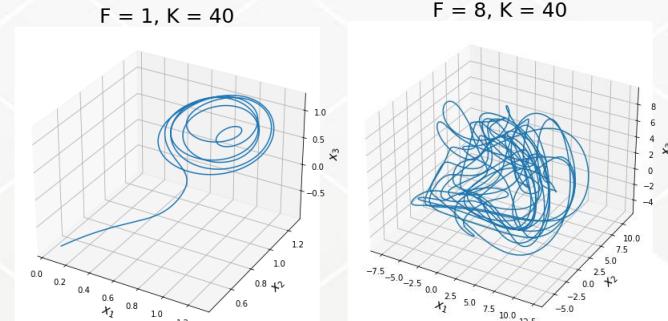
- A dynamical system defined by

$$\frac{dx_k}{dt} = (x_{k+1} - x_{k-2})x_{k-1} - x_k + F$$

where $k = 1, \dots, K$, and $x_{K-K} = x_{K+K} = x_k, K > 3^{[1]}$.



- x_k : values of some atmospheric quantity in K sectors of a latitude circle.
- F and the linear terms simulate the external forcing and internal dissipation.
- Quadratic terms simulate the advection.
- **1 time unit = 5 days** of atmospheric time.
- For the following experiments: $F = 8, K = 40$.

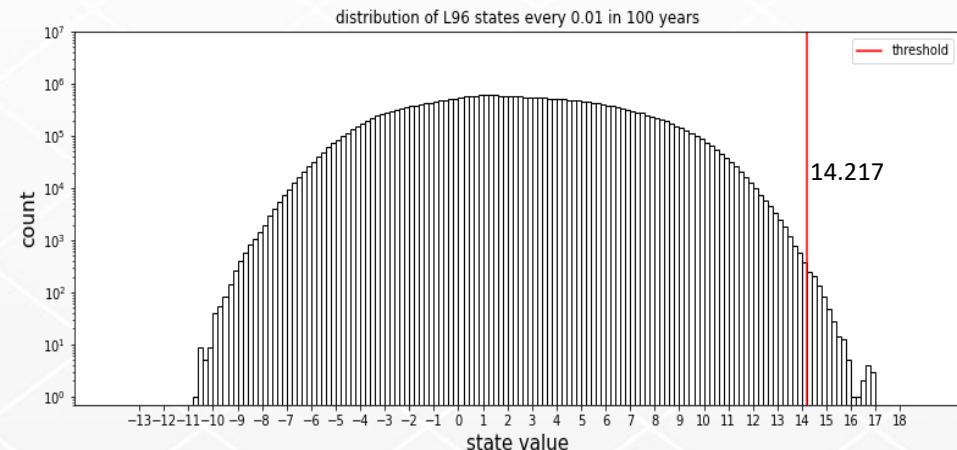
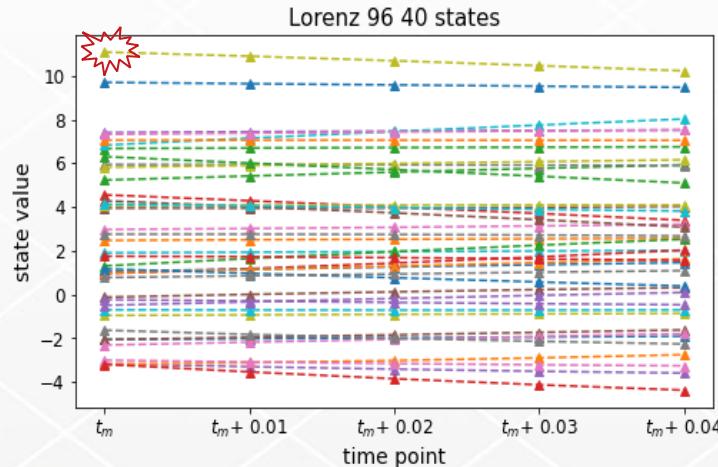


[1] Lorenz, Edward, Predictability – A problem partly solved, *Seminar on Predictability*, Vol. I, ECMWF, 1996.

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Control Experiments - extreme events

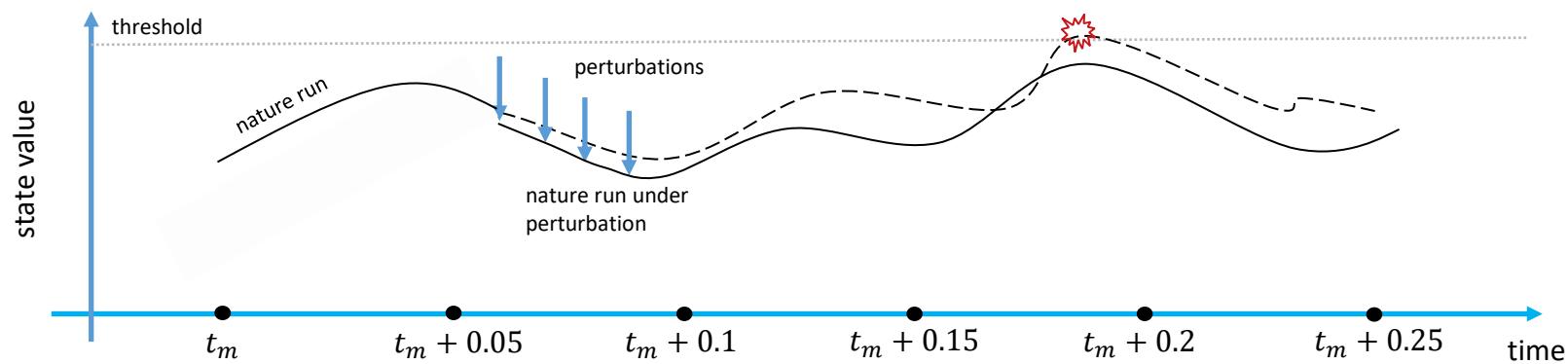
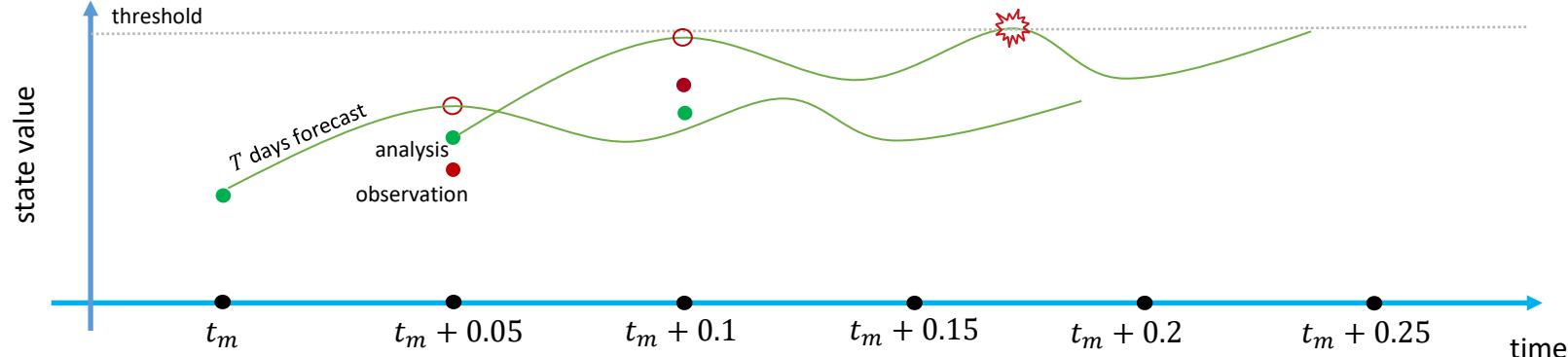
- Aim: to avoid extreme values.
- Integrate L96 over 110 years, keep the record of every $dt=0.01$ for the last 100 years. Record the maximum value over each 6h period.



- The first 200 maximum values are extreme values (on average 2 times / year).

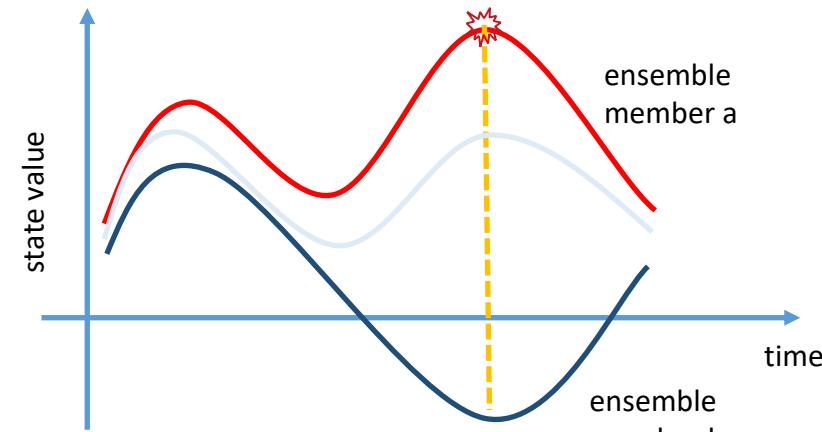
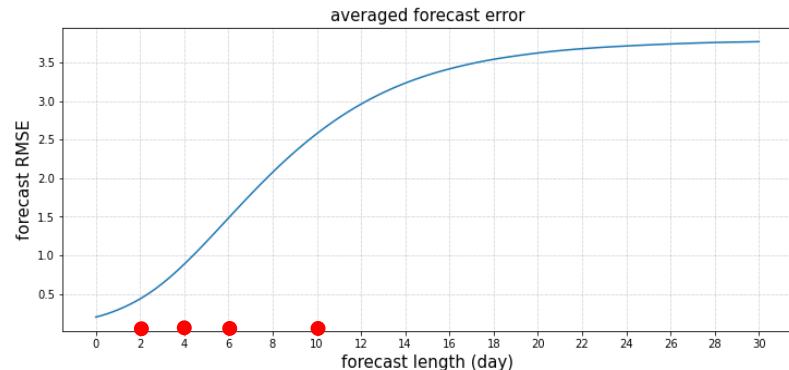
Control Experiments - procedure

- The observations are noised (nature run + $\text{Normal}(0, 1)$).



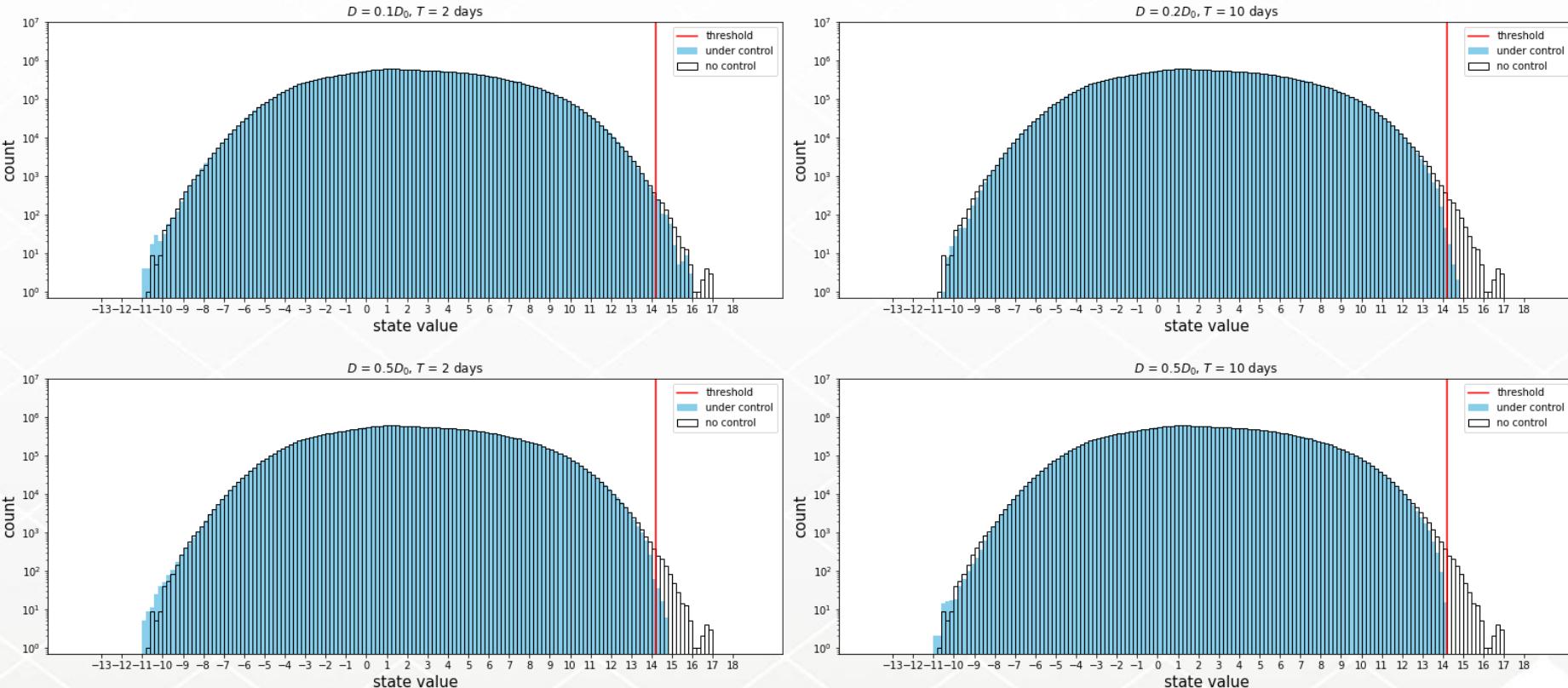
Control Experiments - LETKF

- We use LETKF with 10 ensemble members, $\rho = 1.06$, R-Localization (cut-off radius $2\sqrt{\frac{10}{3}} \times 5.47$) : analysis RMSE $\approx 0.19890 \dots$
- Forecast length T .

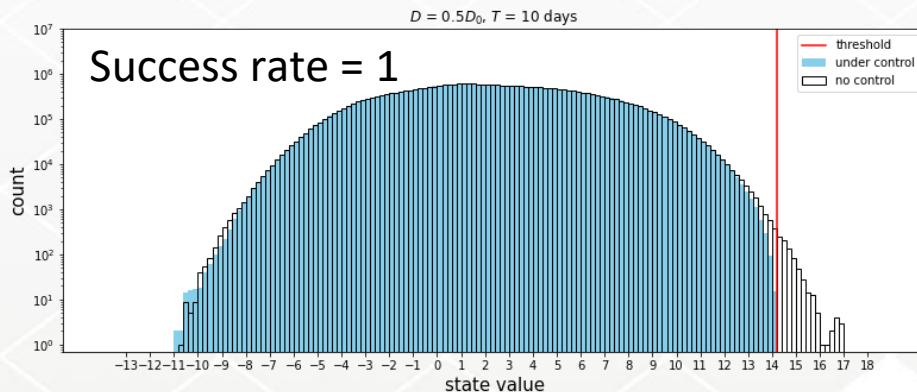
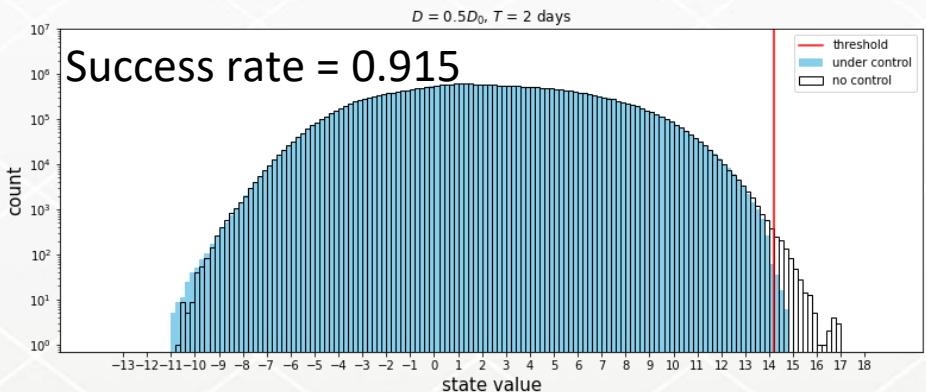
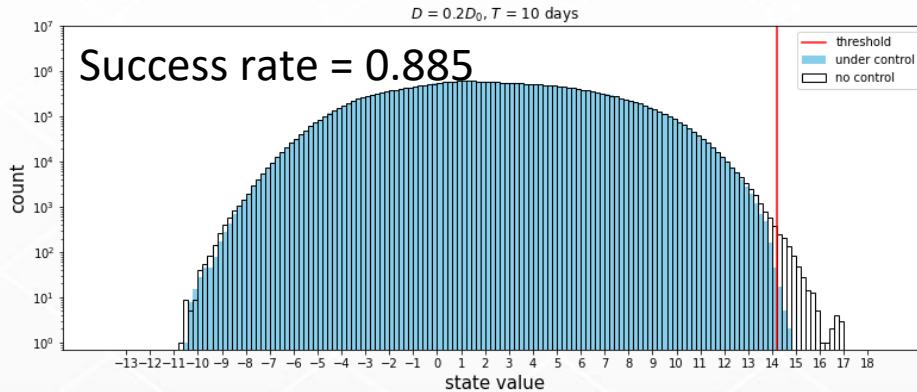
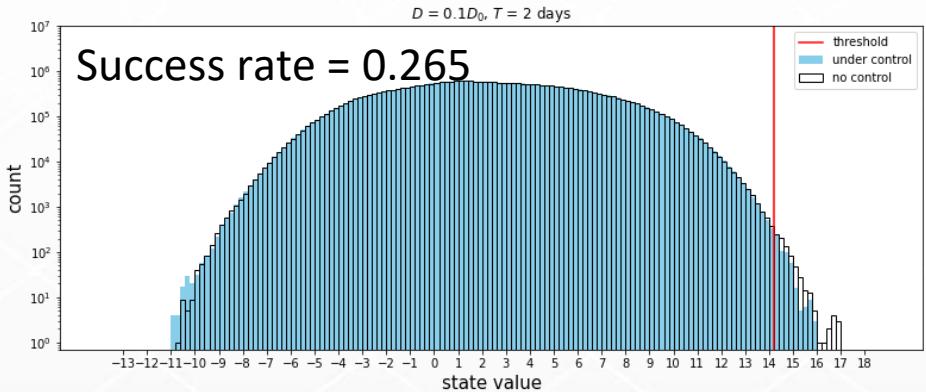


- Control vector: the difference between two proper ensemble members rescaled to a fixed norm.
- Control vector norm: $D = \alpha D_0$ where D_0 is equal to the analysis RMSE.

Control Experiments – results for 100 years



Control Experiments – results for 100 years



Control Experiments – success rate

$T \backslash \alpha$	2 days	4 days	6 days	10 days
0.1	0.42 (0.30, 0.54)	0.63 (0.52, 0.74)	0.8 (0.74, 0.86)	0.84 (0.76, 0.91)
0.2	0.63 (0.57, 0.69)	0.89 (0.86, 0.92)	0.95 (0.93, 0.98)	0.97 (0.94, 0.99)
0.3	0.79 (0.72, 0.86)	0.97 (0.95, 0.99)	0.99 (0.98, 1.00)	0.99 (0.98, 1.00)
0.4	0.84 (0.79, 0.89)	0.98 (0.96, 1.00)	0.99 (0.97, 1.00)	1.00 (0.99, 1.00)
0.5	0.91 (0.86, 0.95)	0.99 (0.97, 1.00)	0.99 (0.99, 1.00)	1.00 (0.99, 1.00)
0.6	0.94 (0.90, 0.97)	0.99 (0.98, 1.00)	1.00 (0.99, 1.00)	1.00 (0.99, 1.00)
0.7	0.95 (0.92, 0.98)	0.99 (0.98, 1.00)	1.00 (1.00, 1.00)	1.00 (1.00, 1.00)
0.8	0.98 (0.96, 0.99)	1.00 (1.00, 1.00)	1.00 (0.99, 1.00)	1.00 (1.00, 1.00)
0.9	0.98 (0.96, 1.00)	1.00 (0.99, 1.00)	1.00 (1.00, 1.00)	1.00 (1.00, 1.00)
1.0	0.98 (0.97, 1.00)	1.00 (0.99, 1.00)	1.00 (0.99, 1.00)	1.00 (1.00, 1.00)

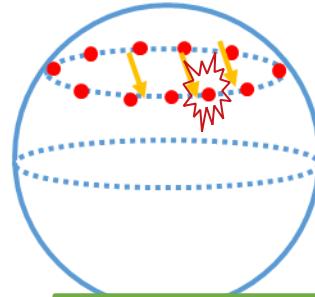
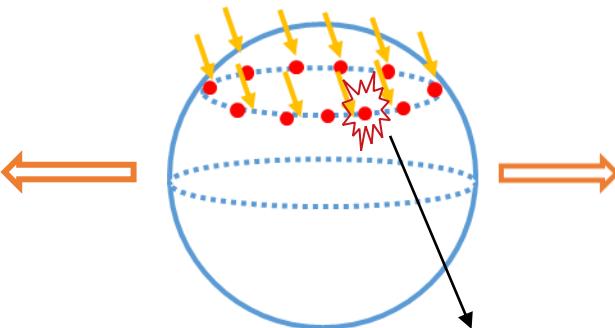
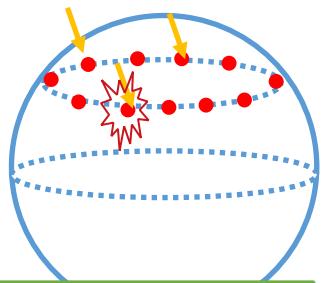
1	[0.9,1)	[0.8,0.9)	[0.6,0.8)	[0.0,0.6)

Control Experiments – energy (Σ 1–norm)

T α	2 days	4 days	6 days	10 days
0.1	$3.98 \times 10^2 \pm 43.5$	$8.91 \times 10^2 \pm 50.5$	$1.62 \times 10^3 \pm 55.6$	$4.08 \times 10^3 \pm 100.5$
0.2	$7.33 \times 10^2 \pm 59.1$	$1.50 \times 10^3 \pm 143.0$	$2.84 \times 10^3 \pm 98.8$	$6.93 \times 10^3 \pm 175.9$
0.3	$1.04 \times 10^3 \pm 101.4$	$1.96 \times 10^3 \pm 165.1$	$3.73 \times 10^3 \pm 167.1$	$9.60 \times 10^3 \pm 278.6$
0.4	$1.28 \times 10^3 \pm 139.1$	$2.44 \times 10^3 \pm 183.6$	$4.55 \times 10^3 \pm 216.8$	$1.19 \times 10^4 \pm 327.3$
0.5	$1.52 \times 10^3 \pm 151.3$	$2.77 \times 10^3 \pm 166.3$	$5.15 \times 10^3 \pm 211.3$	$1.40 \times 10^4 \pm 358.1$
0.6	$1.74 \times 10^3 \pm 132.8$	$3.07 \times 10^3 \pm 272.4$	$5.87 \times 10^3 \pm 183.7$	$1.61 \times 10^4 \pm 467.7$
0.7	$1.95 \times 10^3 \pm 129.6$	$3.34 \times 10^3 \pm 204.0$	$6.40 \times 10^3 \pm 300.5$	$1.81 \times 10^4 \pm 479.6$
0.8	$2.03 \times 10^3 \pm 274.4$	$3.60 \times 10^3 \pm 255.0$	$6.82 \times 10^3 \pm 333.3$	$1.98 \times 10^4 \pm 329.7$
0.9	$2.23 \times 10^3 \pm 152.5$	$3.83 \times 10^3 \pm 197.1$	$7.42 \times 10^3 \pm 216.6$	$2.16 \times 10^4 \pm 590.1$
1.0	$2.32 \times 10^3 \pm 203.1$	$4.13 \times 10^3 \pm 183.3$	$7.74 \times 10^3 \pm 306.5$	$2.34 \times 10^4 \pm 714.8$

≥ 20000	[10000,20000)	[6000,10000)	[3000,6000)	[1000,3000)	≤ 1000

Partial control

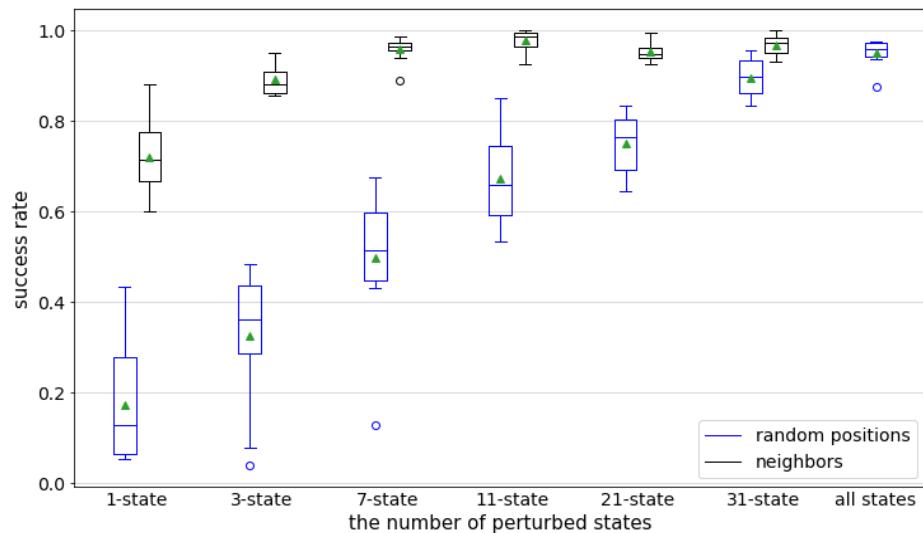
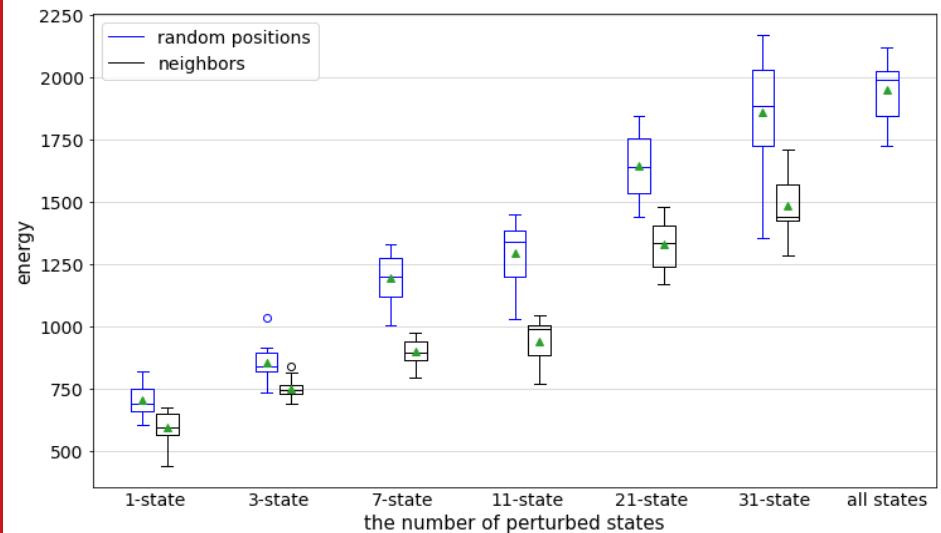


Random select m states.
Perturb these states
when necessary.

State shows the most
extreme value in forecast

Perturb the corresponding
state (and its neighbors)

Partial control



References

- [1] Lorenz, E., 1996: Seminar on Predictability, Vol. I, ECMWF.
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- [4] Houtekamer, P. L., and Zhang, F.: Review of the ensemble Kalman filter for atmospheric data assimilation, Mon. Wea. Rev., 144, 4489-4532, 2016.
- [5] Evensen, G.: Advanced data assimilation for strongly nonlinear dynamics, Mon. Wea. Rev., 125, 1342–1354, 1997.
- [6] Brian R. Hunt, Eric J. Kostelich, Istvan Szunyogh, Efficient data assimilation for spatiotemporal chaos: A local ensemble transform Kalman filter, Physica D: Nonlinear Phenomena, Volume 230, Issues 1–2, 2007, 112-126, ISSN 0167-2789, 2007.

Thank you for your attention!