Abstract

[Mini-School]

Norbert A'Campo (Basel University)

Lagrangian spine and symplectic monodromy

Abstract: Study of isolated of complex hypersurfaces from the symplectic view point. See the paper, still work in progress, "Lagrangian spine and symplectic monodromy" on Conference web page.

Georges Comte (Université Savoie Mont Blanc, Chambéry)

Zeroes lemmas and counting of algebraic points on tame and oscillatory curves

Abstract: I will present old and new results concerning Bézout bounds. More accurately, X being a subset of $\mathbb{R}^n$, a Bézout bound $Z(X,d)$ of X in degree d is a bound for the number of isolated intersection points of X with any algebraic hypersurfaces of degree d. For a given set X, the knowledge of the asymptotics w.r.t. d of $Z(X,d)$ has important applications in Diophantine geometry: following Bombieri-Pila's approach, one can bound in term of $Z(X,d)$ the number of algebraic points of given algebraic degree and given Weil height. In turn, this kind of bounds as been used in the o-minimal context to give a proof of André-Oort conjecture (a generalization of the Mannin-Mumford conjecture) by Pila, Zannier, Ulmo, Yafaev and others mathematicians.

I will present the o-minimal context for counting algebraic points (Pila-Wilkie's theorem), and the case where X is a graph of a holomorphic or a meromorphic function, as well as the case of oscillatory curves.

Mutsuo Oka (Tokyo University of Science)

Introduction to mixed hyper surface singularity

Abstract: Introduction of a basic properties of mixed hypersurfaces will be given and then a comparison with complex analytic singularities will be explained.

[Symposium]

Jean-Paul Brasselet (Aix-Marseille Université, Marseille)

Coincidences

Abstract: This is a join work with Tatsuo Suwa (Hokkaido).

Given two maps f and g between compact oriented manifolds M and N of the same dimension, the coincidence points are defined as points $x \in M$ such that $f(x) = g(x)$. At these points, one defines the local coincidence index (intersection index of the graphs).

Lefschetz coincidence Theorem says that the sum of indices is equal to alternating sum of suitable matrix traces. The Lefschetz fixed point formula is just the case $M = N$ and g is the identity map 1_M on M.

On the one hand, the result has been generalized in the case of manifolds with different dimensions. On the other hand, M. Goresky and R. MacPherson have extended the Lefschetz fixed point theorem for singular varieties in the context of intersection homology and with suitable hypothesis on the varieties and the maps.

In this lecture, I will provide the main definitions and results concerning the Lefschetz coincidence local indices and classes in the case of manifolds with same and possibly different dimensions. This is the contribution due to Tatsuo Suwa and myself.

In the case of possibly singular varieties, I will recall the situation of the Goresky-MacPherson Lefschetz fixed point theorem for singular varieties. That leads to the Lefschetz coincidence Theorem in the case of singular varieties. I will give examples to illustrate the results.

Le Quy Thuong (VNU University of Science)

Motivic Milnor fibers of plane curve singularities

Abstract: We compute the motivic Milnor fiber of a complex plane curve singularity in an inductive and combinatoric way using the extended simplified resolution graph. An application is that one can study the Hodge-Steenbrink spectrum of such a singularity in terms of that of a quasi-homogeneous singularity.

Toshizumi Fukui (Saitama University)

On bifurcation of Euler beam problem

Abstract: The equation for buckling of a rod is obtained by a variational method and its solution has pitchfolk bifurcation. Golubistky and Schaeffer analyzed its perturbed problem and showed that it provides a versal unfolding of the buckling problem. But the bifurcation equation is not analyzed in details. We revisit this problem and investigate the bifurcation locus up to order 3, etc.. We show how the equation depends on the length of the rod. This is a joint work with Atia Afroz.

Miruna-Stefana Sorea (Université Lille 1)

The shapes of level curves of real polynomials near strict local minima

Abstract: We consider a real bivariate polynomial function vanishing at the origin and exhibiting a strict local minimum at this point. We work in a neighbourhood of the origin in which the non-zero level curves of this function are smooth Jordan curves. Whenever the origin is a Morse critical point, the sufficiently small levels become boundaries of convex disks. Otherwise, these level curves may fail to be convex.

The aim of this talk is two-fold. Firstly, to study a combinatorial object measuring this nonconvexity; it is a planar rooted tree. And secondly, we want to characterise all possible topological types of these objects. To this end, we construct a family of polynomial functions with non-Morse strict local minima realising a large class of such trees.

Nguyen Quang Dieu (Ha Noi National University of Education)

Volume estimates for sublevel sets of real and complex polynomials

Abstract: In this talk we will discuss upper bounds for volume of sublevel sets of real and complex polynomials. Our method is to combine a version of global Łojasiewicz inequality with some well known estimates on volume of tubes around real algebraic sets. Some applications to oscillatory integrals and integration indices of real and complex polynomial are also given. This is essentially based on a recent joint work Pham Tien Son, Dau Hoang Hung and Hoang Thieu Anh.

Laurentiu Maxim (University of Wisconsin)

Perverse sheaves on semi-abelian varieties: structure and applications

Abstract: I will present a new characterization of perverse sheaves on complex semi-abelian varieties in terms of their cohomology jump loci, generalizing results of Gabber-Loeser and Schnell. I will also discuss propagation properties and codimension lower bounds for the cohomology jump loci of perverse sheaves. As concrete applications, I will mention: (a) generic vanishing for perverse sheaves on semi-abelian varieties; (b) homological duality properties of complex algebraic manifolds, via abelian duality; (c) new topological characterizations of (semi-)abelian varieties. (Joint work with Y. Liu and B. Wang.)

Takehiro Saito (University of Tsukuba)

Milnor monodromies and mixed Hodge structures for non-isolated hypersurface singularities

Abstract: We study the Milnor monodromies of non-isolated hypersurface singularities. In general, if a singular point is non-isolated, the reduced cohomology groups of the Milnor fiber of the singular point are not concentrated in the middle degree. However, by using the theory of mixed Hodge modules, we can show that the generalized eigenspaces of them for some eigenvalues of the Milnor monodromies are concentrated in the middle degree as in the case for isolated hypersurface singular points. As an application of this result, we can compute some parts of the Jordan normal forms of the Milnor monodromies for non-isolated hypersurface singular points.

Lê Dũng Tráng (ICTP and Hanoi)

A remark on the topology of complex polynomials

Abstract: I shall make a remark on the topology of complex polynomials which has some interest.

Kazushi Ueda (University of Tokyo)

A-infinity algebras and singularities

Abstract: The theory of A-infinity algebras can be regarded as a `formal non-commutative differential graded' generalization of singularity theory. In particular, the Hochschild cohomology of an A-infinity algebra corresponds to the Jacobi ring of a hypersurface singularity, and controls infinitesimal deformations. In the talk, we will discuss the description of the positive part of the base space of the semiuniversal unfolding of an exceptional unimodal singularity as the moduli space of A-infinity structures on a graded algebra. If the time permits, we will also discuss an application to homological mirror symmetry. This is a joint work with Yanki Lekili.

Do Viet Cuong (VNU University of Science)

On the motive of moduli spaces of parabolic Higgs bundles

Abstract: Higgs bundles, introduced by Hitchin, play as a central object of study in geometry with several links to number theory (for example, they appeared in the Ngo's proof for the fundamental lemma for Lie algebras) and physics. Over a connected smooth projective algebraic curve C, we have a correspondence which preserves the rank between the equivalent classes of irreducible representations of $\pi_1(C)$ and the equivalent classes of isomorphisms of stable Higgs bundles.

A natural generalization of the Higgs bundles is the parabolic Higgs bundles (that we shall equip each bundle of a parabolic structure, i.e the choice of flags in the fibers over certain marked points, and some compatible conditions). Simpson proved that there is analogous relation between the representations of the fundamental group of a punctured curve with the parabolic Higgs bundles (the marked points are the points that are took out from the curve).

Despite their good applications, the cohomology of the moduli space of (parabolic) Higgs bundles has not yet been determined. In this talk, I shall explain an algorithm to calculate the (virtual) motive (i.e in a suitable Grothendieck group) of the moduli spaces of ``stable" (parabolic) Higgs bundles. In the case when the moduli space is quasi-projective, the virtual motive allows us to read off the dimensions of its cohomology spaces.

Dominik Wrazidlo (Kyushu University)

The Milnor 7-sphere does not admit a special generic map into $\mathbb{R}^3$

Abstract: In this talk I will present recent progress in the following problem raised by O. Saeki in 1993. Determine the set of integers p for which a given homotopy sphere admits a special generic map into $\mathbb{R}^p$. Here, a so-called special generic map is by definition a map between smooth manifolds all of whose singularities are definite fold points.

By means of the technique of Stein factorization we introduce and study certain special generic maps of homotopy spheres into Euclidean spaces called standard. Modifying a construction due to Weiss, we show that standard special generic maps give naturally rise to a filtration of the group of homotopy spheres by subgroups that is strongly related to the Gromoll filtration. Finally, we apply our result to some concrete homotopy spheres, which in particular answers Saeki's problem for the Milnor 7-sphere.

Masahiko Yoshinaga (Hokkaido University)

G-Tutte polynomials and abelian Lie group arrangements

Abstract: Given a list of integer vectors, one can associate several mathematical objects, e.g., matroids, hyperplane arrangements, toric arrangements, zonotopes etc. Each has certain (quasi)polynomial invariants which possesses rich topological and enumerative information. Among others the Tutte polynomial detects Betti numbers of the complement of a complex hyperplane arrangement. Recently L. Moci introduced the arithmetic Tutte polynomial which acts similarly for toric arrangements. In this talk, we introduce G-Tutte polynomial for an abelian group G (with a weak assumption on the finiteness of torsions). Main examples are abelian Lie groups with finitely many connected components. It is a generalization of "Tutte polynomials" in the sense that $G = \mathbb{C}$ and $\mathbb{C}^*$ recovers Tutte and arithmetic Tutte polynomial, respectively. We see that many well known properties are shared also by G-Tutte polynomials. We also discuss the topology of the complement of corresponding "arrangements" for non-compact group G. This is a joint work with Ye Liu and Tan Nhat Tran (arXiv:1707.04551).

Dinh Si Tiep (Institute of Mathematics, VAST)

Limits of tangent spaces of definable sets

Abstract: One of the ways to study singular varieties is to investigate their tangent cones and limits of tangent spaces, which was initialized by Whitney. For complex analytic varieties, the studies of the set of limits of tangent spaces at a singular point were carried out by Henry, Lê and Teissier. In the real setting, to our knowledge, there is only one case of algebraic surfaces that has already been consider by O'Shea and Wilson ([1]). In this work, by different proofs, we show that the results given in [1] still hold for a much more general context of definable sets of any dimension in any o-minimal structure. This is a joint work with Olivier Le Gal and Tien Son Pham.

[1] D. B. O'Shea, L. C. Wilson, Limits of tangent spaces to real surfaces, Amer. J. Math. 126, no. 5 (2004), 951－980.

Hussein Mourtada (Université Paris 7)

Arc spaces and partition identities

Abstract: We will show a link between the arc space (which is an algebro-geomtric object) and the identities of partitions of integer numbers: a partition of a positive integer number is simply a way of writing it as a sum of positive integer numbers. Integer partitions have a long and beautiful history in number theory. The link that we will describe comes from an invariant of singularities and gives a new point of view on known results and new identities.

Part of a what I will tell is joint work with Pooneh Afsharijoo on one hand and with Clemens Bruschek and Jan Schepers on the other hand.

Tobias Kaiser (Universität Passau)

Hardy Fields, O-minimal Structures, and connections to Hilbert 16

Abstract: In joint work with Zeinab Galal and Patrick Speissegger we have recently constructed a Hardy field which contains all transition maps of plane polynomial vector fields at hyperbolic singularities and all unary functions definable in the o-minimal expansion of the real field by restricted analytic functions and exponentiation. In this

talk I explain these notions and their connections and discuss how o-minimality can make some contributions to questions around Hilbert 16.

Anna Valette (Jagiellonian University, Kraków)

"Nash triviality and generalized critical values" in memory of Masahiro Shiota

Abstract: Let $X \subset \mathbb{R}^n$ be a closed Nash manifold. We show that a Nash mapping $f: X \longrightarrow \mathbb{R}^k$ is a locally Nash trivial outside the set of generalized critical values.

Nguyen Hong Duc (Basque Center for Applied Mathematics)

Equivariant motivic integration and proof of the integral identity conjecture for regular functions

Abstract: We first develop the Denef-Loeser motivic integration to the equivariant motivic integration by using Grothendieck's descending theory. We then apply it to prove the full version of Kontsevich-Soibelman integral identity conjecture for regular functions

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Mickael Matusinski (Université de Bordeaux)

The algebraic closure of formal power series in several variables

Abstract: Joint work in progress with M. Hickel (U. Bordeaux). Our purpose is to understand the algebraic closure of $K((\underline{x}))$, $\underline{x} = (x_1, \dots, x_r)$, inside the field of multivariate Puiseux series with support in some strongly convex rational polyhedral cone (after McDonald, F. Aroca－Ilardi, Soto－Vicente). Our strategy involves the answers that we recently obtained to the same type of questions for algebraic Puiseux series, i.e. for the algebraic closure of $K(\underline{x})$. More precisely, our approach consist in solving the following problems:

- given a polynomial equation $P(\underline{x}, y) = 0$, provide a formula for the coefficients of a Puiseux series solution $y(\underline{x})$ in terms of the coefficients of the equation;
- given an algebraic Puiseux series, reconstruct the coefficients of a vanishing polynomial using the first coefficients of the series.

Tran Nhat Tan (Hokkaido University)

Interpretations for characteristic quasi-polynomials via subspace and toric arrangements

Abstract: Given a central hyperplane arrangement with integral coefficients, Kamiya-Takemura-Terao proved that the cardinality of the complement of the q-reduced arrangement is a quasi-polynomial in q. They called it the characteristic quasi-polynomial as its first constituent coincides with the hyperplane arrangement's characteristic polynomial, and left the task of understanding the other constituents to be an interesting problem. In this talk, we present two interpretations for the characteristic quasi-polynomial and its constituents through subspace and toric arrangement viewpoints. The interpretations are inspired by an observation that the first constituent has the smallest coefficients, those of the last constituent are biggest. Then we find the subspace and toric arrangements essential tools, from which we derive two corresponding methods: adding and removing. This talk contains a recent joint work with Masahiko Yoshinaga.

Hironobu Naoe (Tohoku University)

Lefschetz fibrations of divides and shadows

Abstract: A'Campo proved that the link of any connected divide is fibered, and it was extended by Ishikawa to the case of divides on orientable surfaces. Such a fibration comes from the Lefschetz fibration that canonically corresponds to a divide. The notion of shadow was introduced by Turaev. A shadowed polyhedron is a 2-dimensional polyhedron whose regions are colored by half-integers. Turaev showed that a 4-manifold is reconstructed from a given shadowed polyhedron uniquely (up to diffeomorphism). We give a method for constructing a shadowed polyhedron from a divide. The 4-manifold reconstructed from a shadowed polyhedron admits a Lefschetz fibration if it satisfies a certain property, which we call the LF-property. We will show that the shadowed polyhedron constructed from a divide satisfies this property and the Lefschetz fibration of this polyhedron is isomorphic to that of the divide. Furthermore, applying the same technique to certain free divides we will show that the links of those free divides are fibered with positive monodromy. This is a joint work with Masaharu Ishikawa

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Hiraku Kawanoue (Kyoto University, RIMS)

Embedded resolution of surfaces in positive characteristic

Abstract: I will talk about Idealistic Filtration Program (IFP). IFP is one of approaches for resolution of singularities in positive characteristic, which I proposed and develop jointly with Kenji Matsuki (Purdue). I will review the proof of resolution in characteristic zero briefly and then sketch the ideas of IFP and how it gives surface resolution.

Le Cong Trinh (Qui NhonUniversity)

On tangent cones of algebraic/analytic sets and applications

Abstract: In the first part of the talk I will characterize and compare the *algebraic* and the *geometric* tangent cones of (complex and real) analytic sets. Next, in the second part I will study the *tangent cones at infinity* of complex algebraic sets. Finally, I will present in the third part of the talk the second and higher order tangent cones of algebraic/analytic sets which have many applications, e.g. in Optimization.

Markus J. Pflaum (University of Colorado)

Invariant Whitney Functions

Abstract: A theorem of Gerald Schwarz says that for a linear action of a compact Lie group G on a finite dimensional real vector space V any smooth G-invariant function on V can be written as a composite with the Hilbert map. We prove a similar statement for the case of Whitney functions along a subanalytic set $Z \subset V$ fulfilling some regularity assumptions. In order to deal with the case when Z is not G-stable we use the language of groupoids. This is joint work with H.-Ch. Herbig, Rio de Janeiro.

Nguyen Huu Kien (Université Lille 1 and Hanoi National University of Education)

Upper bound of exponential sums modulo p^m of non-degenerate polynomials

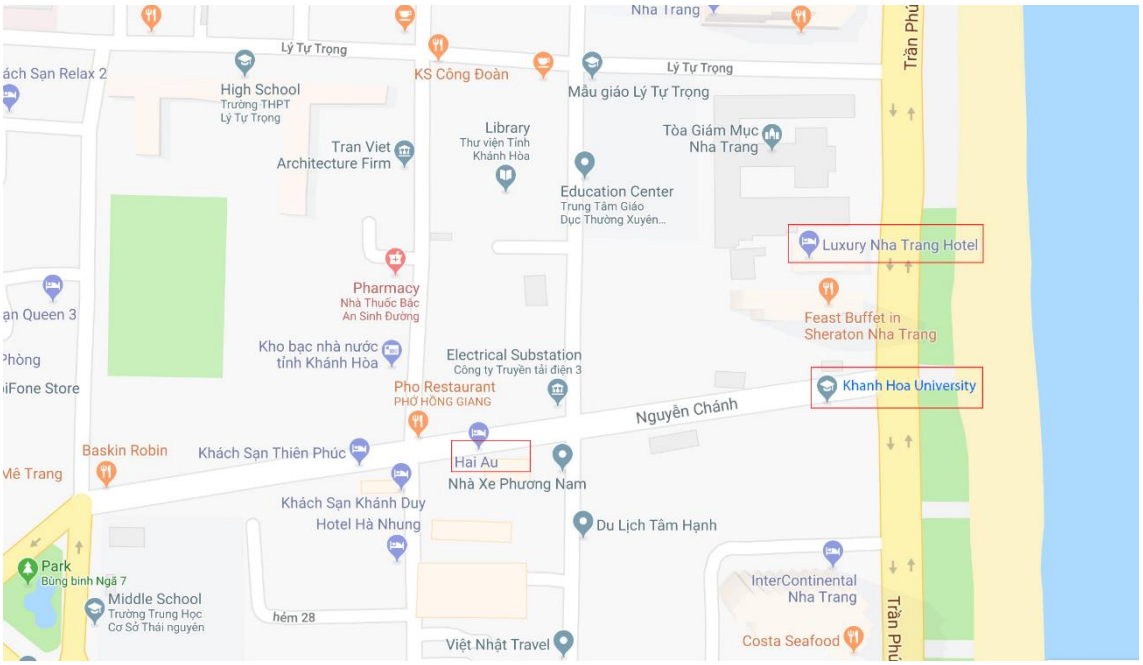
Abstract: This talk is a joint work with Wouter Castryck. Exponential sums modulo p^m of a polynomial was investigated by Igusa in his famous lecture. Igusa conjectured a uniform upper bound on p and m of the global exponential sums modulo p^m with respect to a non-constant homogeneous polynomial with some conditions on the

numerical data given by an embedded resolution of singularities associated with such polynomial. Denef and Sperber considered the conjecture of Igusa for homogeneous polynomial which is non-degenerate with respect to its Newton polyhedron at the origin. With some combinatoric properties of such Newton polyhedron, Denef and Sperber gave a bound of exponential sums depended on Newton polyhedron. Cluckers extended the result of Denef and Sperber to non-degenerate quasi-homogeneous polynomials with no condition on the Newton polyhedron. In this talk we will extend the result of Denef-Sperber and Cluckers to all nondegenerate polynomials. We also use this result to prove a conjecture of Denef and Hoornaert about the candidate-leading Taylor coefficient of Igusa's local zeta function associated to a non-degenerate polynomial, at its largest non-trivial real candidate pole.

Nicolas Dutertre (Université Angers)

Generalizations of the global Euler obstruction and critical points

Abstract: Seade, Tibar and Verjovsky (Math. Annalen, 2005) defined the global Euler obstruction of an affine complex algebraic set and they gave a polar multiplicity formula for this obstruction. In this talk, we define several generalizations of the global Euler obstruction and give several generalizations of the polar multiplicity formula. This is a joint work with Nivaldo Grulha.



(*) The symposium place is **Auditorium 206C** in Khanh Hoa University