

# Equidistribution of non-pluripolar products for holomorphic endomorphisms of $\mathbb{P}^k$

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In this talk, we discuss limit currents of a sequence of normalized pull-backs of positive closed currents by a non-invertible holomorphic endomorphism  $f$  of  $\mathbb{P}^k$ . We first discuss known results. Next we study the notion of non-pluripolar products and then give a motivation of this work and an idea of the proof for the equidistribution of the non-pluripolar products associated with plurisubharmonic functions in the so-called Cegrell class, with exponentially fast convergence.

## Lyapunov exponent of singular holomorphic foliations by curves

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Let  $\mathcal{F}$  be a holomorphic foliation by Riemann surfaces defined on a compact complex projective surface  $X$  satisfying the following two conditions:

- the singular points of  $\mathcal{F}$  are all hyperbolic;
- $\mathcal{F}$  is Brody hyperbolic.

Then we establish cohomological formulas for the Lyapunov exponent and the Poincaré mass of an extremal positive  $\partial\bar{\partial}$ -closed current tangent to  $\mathcal{F}$ .

If, moreover, there is no nonzero positive closed current tangent to  $\mathcal{F}$ , then we show that the Lyapunov exponent  $\lambda(\mathcal{F})$  of  $\mathcal{F}$ , which is, by definition, the Lyapunov exponent of the unique normalized positive  $\partial\bar{\partial}$ -closed current tangent to  $\mathcal{F}$ , is a strictly negative real number.

As an application, we compute the Lyapunov exponent of a generic foliation with a given degree in  $\mathbb{P}^2$ .

## Data assimilation for three-dimensional Navier-Stokes-Voigt equations

Cung The Anh

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We study a nudging data assimilation algorithm for the three-dimensional Navier-Stokes-Voigt equations. Under suitable conditions on the relaxation (nudging) parameter, the spatial mesh resolution, and the time step between successive measurements, we obtain an asymptotic in time estimate of the difference between the

approximating solution and the unknown reference solution corresponding to the measurements, in an appropriate norm, which shows exponential convergence. We will focus on two cases, where measurements of only two components of the velocity field are given or measurement errors are represented by stochastic noises.

## Bubbling of the $Q$ -curvature flow on 4-manifolds

Ngo Quoc Anh

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Geometric flows such as Calabi's flow, Ricci's flow, curvature flow, have been long known to be useful to produce geometric quantities. However, like any heat flow, geometric flows are very sensitive as they can produce blow-up in a complicated and mysterious way. In this talk, I will describe a potential mechanism to handle bubbling phenomena for higher-order curvature flows via an example of the  $Q$ -curvature flow on 4-manifolds.

This talk is based on joint work, announced at arXiv:1903.12054, with Hong Zhang from USTC in China

## On Eigenvalues Problems for Degenerate Elliptic Operators with Non-smooth Coefficients

Hua Chen

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Let  $n \geq 2$ ,  $X = (X_1, X_2, \dots, X_n)$  are real vector fields defined in  $\mathbb{R}^n$ . For each  $X_k$ , we assume that  $X_k = \mu_k(x)\partial_{x_k}$ , where  $\mu_1 = 1$ , and for  $2 \leq j \leq n$ ,  $\mu_j(x) = \mu_j(x_1, \dots, x_{j-1}) \in C^1(\mathbb{R}^n \setminus \Pi)$  are real continuous nonnegative functions with  $\Pi = \left\{ (x_1, \dots, x_n) \in \mathbb{R}^n \mid \prod_{i=1}^n x_i = 0 \right\}$ . Suppose that  $\Omega$  is a bounded open subset in  $\mathbb{R}^n$  with smooth boundary  $\partial\Omega$  and  $\Omega \cap \Pi \neq \emptyset$ .

We introduce the degenerate  $\Delta_\mu$ -Laplace operator as  $\Delta_\mu := -\sum_{j=1}^n X_j^* X_j$ . Denote the weighted Sobolev space  $H_X^1(\Omega) = \{u \in L^2(\Omega) \mid X_j u \in L^2(\Omega), j = 1, \dots, n\}$ . Then  $H_X^1(\Omega)$  is a Hilbert space endowed with norm  $\|u\|_{H_X^1}^2 = \|u\|_{L^2(\Omega)}^2 + \|Xu\|_{L^2(\Omega)}^2$ , where  $\|Xu\|_{L^2}^2 = \sum_{j=1}^n \|X_j u\|_{L^2}^2$ .

Now, we consider the following Dirichlet eigenvalue problem of  $-\Delta_\mu$  in  $H_{X,0}^1(\Omega)$

$$\begin{cases} -\Delta_\mu u = \lambda u, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases}$$

where  $H_{X,0}^1(\Omega)$  is the closure of  $C_0^\infty(\Omega)$  in  $H_X^1(\Omega)$ . Then the self-adjoint operator  $-\Delta_\mu$  has discrete Dirichlet eigenvalues  $0 < \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_{k-1} \leq \lambda_k \leq \dots$ , and  $\lambda_k \rightarrow +\infty$  as  $k \rightarrow +\infty$ .

In this talk, under some conditions, we shall give sharp lower bounds and upper bounds for Dirichlet eigenvalues  $\lambda_k$ .

# Embedding theorems for weighted fractional Sobolev spaces and their applications

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Let  $\Omega$  be a reasonable nice open set (bounded or unbounded) in  $\mathbb{R}^n$ . Let  $\rho$  be a positive locally integrable function on  $\Omega$  such that it is essentially constant on each ball that is deep inside  $\Omega \setminus F$  where  $F$  is a finite collections of points or hyperplanes. Let  $\mu, w, \sigma$  be Borel measures on  $\Omega$ . Combining some previously obtained results, we study embedding and compact embedding theorems of weighted fractional Sobolev spaces  $W_{w\rho}^{s,p}(\Omega)$  to  $L_{\mu\rho}^q(\Omega)$  where

$$W_{\rho w}^{s,p}(\Omega) = \{f \in L_{loc,w}^1(\Omega) : \|f\|_{W_{\rho w}^{s,p}(\Omega)} < \infty\} \quad 0 < s < 1$$

$$\|f\|_{W_{\rho w}^{s,p}(\Omega)} = \left( \int_{\Omega} \int_{\Omega} \frac{|f(x) - f(y)|^p}{|x - y|^{n+sp}} \rho(x)\rho(y)dw(x)dw(y) \right)^{1/p}.$$

We will extend a result of Bourgain et al in 2002 that relates unweighted fractional Sobolev spaces  $W^{s,p}$  to the usual 1st order Sobolev space  $W^{1,p}$  as  $s \rightarrow 1^-$  on cubes to John domains. We also provide some simple application of our results to some PDE of nonlocal operators such as

$$\int_{\mathbb{R}^n} \int_{\mathbb{R}^n} \frac{(u(x) - u(y))(\phi(x) - \phi(y))}{|x - y|^{n+2s}} d\sigma(x)d\sigma(y) = \int_{\mathbb{R}^n} G(x, f)\phi(x)d\sigma(x)$$

for all test functions  $\phi$ .

Note that the above is an extension of the following equation arising from Lévy process

$$(-\Delta)^s u(x) = G(u, x) \quad \text{on } \Omega \quad \text{and } u = 0 \text{ outside } \Omega,$$

where the fractional Laplacian can be written as

$$(-\Delta)^s u(x) = C(n, s) \text{PV} \int_{\mathbb{R}^n} \frac{u(x) - u(y)}{|x - y|^{n+2s}} dy.$$

## Exponential dichotomy and inhomogeneous evolution equations on the line

Trinh Viet Duoc

VNU University of Science

In the talk, we show that the exponential dichotomy of a strongly continuous invertible evolution family on a Banach space is equivalent to the existence and uniqueness of admissible solution of the corresponding inhomogeneous evolution equation.

# On regularity of weak solutions for the Navier-Stokes equations in general domains

Vu Thi Thuy Duong

Quang Ninh University of Industry

Let  $u$  be a weak solution of the instationary Navier-Stokes equations in a completely general domain  $\Omega \subseteq \mathbb{R}^3$  which additionally satisfies the strong energy inequality. Firstly, we prove that  $u$  is regular if the kinetic energy  $\frac{1}{2} \|u(t)\|_2^2$  is left-side Holder continuous with Holder exponent and with a sufficiently small Holder seminorm. This result extends the previous ones by several authors in which the domain  $\Omega$  is additionally supposed to be bounded. Secondly, we show that if  $u(t) \in D(A^{1/4})$  and

$$\lim_{\delta \rightarrow 0^+} \|A^{1/4}(u(t-\delta) - u(t))\|_2 < C, \text{ for all } t \in [0, T)$$

with a sufficiently small positive constant  $C$  then  $u$  is regular in  $[0, T)$ . Our proofs use the theory about the existence of local strong solutions and uniqueness arguments in the general domain.

Joint work with D. Q. Khai, and N. M. Tri.

## Large derivations theorem for holomorphic endomorphisms of $\mathbb{P}^k$

Wu Hao

National University of Singapore, Singapore

Let  $f$  be a non-invertible holomorphic endomorphism of the complex projective space  $\mathbb{P}^k$ . Let  $\mu$  be the equilibrium measure of  $f$ . I will introduce a large derivations theorem for  $\mu$  and for bounded d.s.h. functions and a large derivations theorem for all d.s.h. functions.

## Stein Spaces with spherical boundary, Generalized Qi-Keng Lu's theorem for Stein space and S. Y. Cheng's conjecture

Xiaojun Huang

Rutgers university, U.S.A

I discuss a joint work with Ming Xiao from UCSD on Stein spaces with spherical boundaries and an application to the S. Y. Cheng's conjecture on Bergman-Einstein metrics.

# On a sharp Sobolev inequality in spaces associated with degenerate elliptic

Nguyen Thi Thu Huong

Hanoi University of Science and Technology

In this talk, we introduce a viscosity approach for the Dirichlet problem associated to complex Hessian type equations on In this talk I present about a sharp Sobolev inequality whose extremal function is a positive solution to a critical semilinear elliptic equation involving degenerate elliptic differential operators. Our result generalizes those obtained in [1].

Joint work with Prof. N. M. Tri, Institute of Mathematics.

## References

- [1] W. Beckner, *On the Grushin operator and hyperbolic geometry*, Proc. of AMS, **129**, No. 4 (1999): 1233–1246.

# Dynamics of correspondences on Riemann Surfaces

Lucas Kaufmann

National University of Singapore

Let  $X$  be a compact Riemann surface. A holomorphic correspondence  $f$  on  $X$  is a multi-valued holomorphic map from  $X$  to itself. Each point of  $X$  has  $d$  images and  $d'$  pre-images counting multiplicity. As in the case of maps, we can iterate  $f$  and study its dynamics. When  $d$  and  $d'$  are different the global dynamics of  $f$  is well understood and  $f$  admits a canonical invariant probability measure.

In this talk I will present some results concerning the case  $d = d'$ . We show that, under a mild and necessary condition that we call non weak modularity,  $f$  admits two canonical probability measures  $\mu^+$  and  $\mu^-$  which are invariant by  $f^*$  and  $f_*$  respectively. These measures enjoy many good properties and describe the distribution of images and pre-images of a point by  $f$ .

As an application, we can consider group actions on the Riemann sphere and recover some classical results about random products of matrices. This is joint work with T.-C. Dinh and H. Wu.

## Resonances for large random samples

Frederic Klopp

Mathematics Institute of Jussieu–Paris Rive Gauche, France

The talk is devoted to the description of the resonances generated by a large sample of random material. In one dimension, one obtains a very precise description for the resonances that directly related to the description for the eigenvalues and localization centers for the full random model. In higher dimensions, below a region of strong localization for the full random model, one computes the asymptotic density of resonances in some exponentially small strip neighboring the real axis.

The talk is partially based on joint work with M. Vogel.

## The constant appeared in the arithmetic general theorem of Ru-Vojta

Min Ru

University of Houston, U.S.A

It is noted, mainly by Nathan Grieve, that the constant appeared in the arithmetic general theorem of Ru-Vojta is related to the (asymptotic) “log canonical threshold” of the line bundle  $L$ .

It is thus related to the  $K$ -stability in the Kahler geometry. In this talk, I’ll try to describe some of its connections.

## On exponential dichotomy and stable manifolds for differential-algebraic equations on the half-line

Ha Phi

College of Science, VNU University of Science

In this work we study linear/semi-linear differential-algebraic equations (DAEs) on the half-line  $\mathbb{R}_+$ . Firstly, we characterize the existence of exponential dichotomy for linear DAEs by invoking the Lyapunov-Perron method. Then, we prove the existence of (local and global) stable manifolds for semi-linear DAEs in the case that the corresponding evolution family to an associated linear DAE has exponential dichotomy, and in addition, an inhomogeneity function fulfills the non-uniform  $\varphi$ -Lipschitz condition.

# On the Chern-Moser-Weyl tensor of real hypersurfaces

Duong Ngoc Son  
University of Vienna, Austria

In this talk, I will discuss a new explicit formula for the well-known Chern-Moser-Weyl tensor for nondegenerate real hypersurfaces in complex space in terms of their defining functions. The formula is considerably simplified when applying to "pluri-harmonic perturbations" of the sphere or to a Fefferman approximate solution to the complex Monge-Ampère equation. The formula is used to show that the  $CR$  invariant one-form recently constructed by Case and Gover is nontrivial on each real ellipsoid of revolution in complex space of three dimension, unless it is equivalent to the sphere. This resolves affirmatively a question posed by these two authors in 2017 regarding the (non-) local  $CR$  invariance of the I-prime pseudohermitian invariant in dimension five and hence provides a counterexample to a recent conjecture by Hirachi. This is a joint work with Michael Reiter.

## A viscosity approach to the Dirichlet problem for degenerate complex Hessian type equations

Do Hoang Son  
Institute of Mathematics, VAST

In this talk, we introduce a viscosity approach for the Dirichlet problem associated to complex Hessian type equations on domains in  $C^n$ . We define a general notion of viscosity sub and supersolutions for which we prove an important comparison principle. Using the Perron method and the comparison principle, we prove the existence of continuous solutions provided bounded sub and supersolutions exist. We also establish a result about the Holder continuity of solution. Finally, we show relations between viscosity concepts and pluripotential concepts in the case of the inverse.

## Compactness for the space of Kahler-Einstein manifolds of negative scalar curvature

Jian Song  
Rutgers University, U.S.A

Let  $K(n, V)$  be the set of  $n$ -dimensional compact Kahler-Einstein manifolds  $(X, g)$  satisfying  $Ric(g) = -g$  with volume bounded above by  $V$ . Then any sequence in  $K(n, V)$  converge, after passing to a subsequence, in pointed Gromov-Hausdorff topology, to a finite union of complete Kahler-Einstein metric spaces. The convergence is smooth off a closed singular set of Hausdorff dimension no greater than  $2n-4$ . Furthermore, the limiting metric space is biholomorphic to an  $n$ -dimensional semi-log

canonical model with its non log terminal locus removed. Our result is a high dimensional generalization for the compactness of constant curvature metrics on Riemann surfaces of genus greater than one. We will also discuss some applications.

## **Equivalence of Cauchy-Riemann manifolds and multisummability theory**

Laurent Stolovitch

University of Nice Sophia Antipolis, France

We prove that if two real-analytic hypersurfaces in  $\mathbb{C}^2$  are equivalent formally, then they are also  $C^\infty$  CR-equivalent at the respective point. As a corollary, we prove that all formal equivalences between real-algebraic Levi-nonflat hypersurfaces in  $\mathbb{C}^2$  are algebraic (in particular are convergent). The result is obtained by using the recent CR - DS technique, connecting degenerate CR-manifolds and Dynamical Systems, and employing subsequently the multisummability theory of divergent power series used in the Dynamical Systems theory. This is a joint work with I. Kossovskiy and B. Lamel.

## **Non-integrated defect of meromorphic maps on Kahler manifolds**

Do Duc Thai

Hanoi University of Education, Vietnam

The purpose of this talk is twofold. The first is to establish a truncated non-integrated defect relation for meromorphic mappings from a complete Kahler manifold quotient of a ball into a projective variety intersecting hypersurfaces in subgeneral position. We also apply it to the Gauss mapping from a closed regular submanifold of  $\mathbb{C}^m$ . The second aim is to establish an above type theorem with truncation level 1 for differentially nondegenerate meromorphic mappings.

This is a work joining with Si Duc Quang.

## **Dispersive estimates for self-adjoint operators on metric measure spaces**

Duong Xuan Thinh

Macquarie University, Australia

Let  $X$  be a metric space with a doubling measure satisfying  $\mu(B) \gtrsim r_B^n$  for any ball  $B$  with any radius  $r_B > 0$ . Let  $L$  be a non negative selfadjoint operator on  $L^2(X)$ .

We assume that the semigroup  $e^{-tL}$  satisfies a Gaussian upper bound and that the flow  $e^{itL}$  satisfies a typical  $L^1 - L^\infty$  dispersive estimate of the form

$$\|e^{itL}\|_{L^1 \rightarrow L^\infty} \lesssim |t|^{-\frac{n}{2}}.$$

Then we prove a similar  $L^1 - L^\infty$  dispersive estimate for a general class of flows  $e^{it\phi(L)}$ , with  $\phi(r)$  of power type near 0 and near  $\infty$ . In the case of fractional powers  $\phi(L) = L^\nu$ ,  $\nu \in (0, 1)$ , we deduce dispersive estimates for  $e^{itL^\nu}$  with data in Sobolev, Besov or Hardy spaces  $H_L^p$  with  $p \in (0, 1]$ , associated to the operator  $L$ .

This is joint work with Piero D'Ancona, The Anh Bui and Detlef Müller.

## Characterization of models in $\mathbb{C}^n$ by their automorphism groups

Ninh Van Thu

VNU University of Science

In this talk, we consider an  $h$ -extendible domain  $\Omega \subset \mathbb{C}^{n+1}$  with noncompact automorphism group. The purpose of this talk is to prove that  $\Omega$  is biholomorphically equivalent to a domain of the form

$$M_P = \{(z, w) \in \mathbb{C}^n \times \mathbb{C} : \operatorname{Re}(w) + P(z) < 0\},$$

where  $P$  is a  $(1/m_1, \dots, 1/m_n)$ -homogeneous plurisubharmonic real-valued polynomial containing no pluriharmonic monomials.

# Some basic ergodic properties for meromorphic maps

Truong Trung Tuyen  
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For a continuous map on a compact metric space, there are basic ergodic properties such as existence of invariant measures and the Variational principle. One important property that makes this possible is that we can push forward a probability measure and obtain again a probability measure (and dually pullback a continuous function and obtain again a continuous function of the same mass norm). This is no longer true for meromorphic maps, because of the existence of indeterminacy points. In this talk, I will present a way to overcome this, by working with an object more general than a measure, so-called strong submeasures. The classical way of pullback a pluri-subharmonic function by a meromorphic map is generalised in this treatment to pullback of a continuous function.

Strong submeasures are relevant in view of the fact that there is now some evidence for that a major folklore conjecture in complex dynamics in higher dimensions is not true [work in progress with Cinzia Bisi]. Also, strong submeasures can be used to define intersection of positive closed  $(1, 1)$  currents on compact Kahler manifolds.

## Asymptotics for the focusing integrable discrete nonlinear Schrödinger equation

Hideshi Yamane  
Kwansei Gakuin University, Japan

The focusing integrable discrete nonlinear Schrödinger equation can be solved by inverse scattering. In the reflectionless case, it admits a (multi-)soliton solution under reasonable assumptions. Phase shifts are determined by the eigenvalues. We consider what happens if the reflection coefficient does not vanish identically. The soliton resolution conjecture is valid. Namely, the solution is asymptotically a sum of 1-solitons. If  $|n/t|$  is less than 2, the phase shifts depend on the eigenvalues and the reflection coefficient. If  $|n/t|$  is not less than 2, they are independent of the reflection coefficient.